A Metric of Elongation
With a Program for Computation

by
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The distance between two triangle centroids varies with the elongation of the containing triangle.

The first such centroid may be termed the BARYCENTER and is the point determined by the respective arithmetic means of the x co-ordinates and the y co-ordinates.

That is the point (x_v,y_v) where:-

$$x_v = \frac{\sum_{i=1}^{3} x_i}{3} \qquad \textbf{Eqn.1}$$

$$y_v = \frac{\sum_{i=1}^{3} y_i}{3} \qquad \textbf{Eqn.2}$$

This is identical with the mechanical center of gravity of a triangular lamina.

The second centroid may be styled the GONIOCENTER and is the point of intersection of the three internal bisectors of the triangle. In practical terms it is of course only necessary to employ two of these bisectors. Such a bisector may be computed from the normal equations of the two flanking sides using the relation:-

$$\frac{A_1 x + B_1 y + C_1}{\sqrt{A_1^2 + B_1^2}} = \pm \frac{A_2 x + B_2 y + C_2}{\sqrt{A_2^2 + B_2^2}} \qquad \textbf{Eqn.3}$$

where the ± allows for the fact that the produced sides form a chiasm with two orthogonal bisectors: The intercept of the required bisector is included between the numeric values of the intercepts of the two sides.

The distance between the BARYCENTER and the GONIOCENTER is the SEPARATION, δ, which varies from zero for equilateral triangles to plus infinity for very extended isosceles or scalene triangles. In order to have a useful metric of extension we may define ELONGATION, η, to be:-

$$\eta = \frac{6\delta}{P} \qquad \textbf{Eqn.4}$$

which varies from zero for equilateral triangles to unity for very extended isosceles or scalene triangles.

In terms of simple side length ratios, η displays asymptotic behavior. In order to recover linearity on the interval 0 to +∞ we may employ ARITHMETISED ELONGATION, γ, defined as:-

$$\gamma = \frac{2}{-\log_n \eta}$$ **Eqn.5**

The Determination of the Bisector Gradients

Note that by definition:-

$$b_l = -\frac{A_l}{B_l}$$ **Eqn.6**

whilst a convenient, and contrasting, property of the coefficient differences is that:-

$$\beta_k = \frac{v_k}{u_k}$$ **Eqn.7**

which leads directly to the slopes of the lines which cross at the goniocenter.

Because the analytic bisectors are paired orthogonals, it may occasionally be necessary to employ the bisector normal to that computed. This condition is tested by checking whether the bisector's intercept falls outwith the range projected by the including sides' intercepts.

This Bisector Intercept, α_k, can be acquired using:-

$$\alpha_k = y_i - \beta_k . x_i$$ **Eqn.8**

Under such circumstances the Bisector Gradient, β_k, is reset using:-

$$\beta_k = -\frac{1}{\beta_k}$$ **Eqn.9**

The Determination of the Goniocenter Co-ordinates

These are found by simultaneous solution of the bisectors' normal equations to establish the common point (x_g,y_g), the goniocenter.

The convenient equations are:-

$$x_g = \frac{\beta_1 x_1 - \beta_2 x_3 - y_1 + y_3}{\beta_1 - \beta_2}$$ **Eqn.10**

$$y_g = \beta_1 x \ (x_g - x_1) + y_1$$ **Eqn.10a**

An Algorithm for the Goniocenter

A robust algorithm requires that:-

(a) The vertices of the triangle are sorted in terms of the x co-ordinates

(b) No co-ordinate has a value equal to another

A simple bubble sort certifies (a) whilst (b) can, if necessary, be obtained by rigidly rotating the second and third vertices about the first through some small arbitrary angle.

The QBasic program ELONGA.BAS of Appendix One realises these preparatory procedures as Subroutine VERTEXSORT.

The goniocentric determination proper is achieved by Subroutine GONIOCENTER which essentially uses two applications of Equation Three rearranged as:-

$$\sqrt{A_2^2 + B_2^2}(A_1 x + B_1 y + C_1) = \sqrt{A_1^2 + B_1^2}(A_2 x + B_2 y + C_2) \qquad \textbf{Eqn.11}$$

to yield a bisector of sides AB/AC via development of the difference of the sides' normal equations (i.e. the bisector normal equations); followed by a similar treatment of sides AC/BC; and finished by a simultaneous equation solution for x_g,y_g based upon the normal equations of the bisectors.

The sequential scheme is:-

(A) Define Edges as First Degree Polynomials according to:-

$$y = a + bx \qquad \textbf{Eqn.12}$$

An incidental benefit is that all y coefficients commence as unity.

(B) Define the Normal Equation Coefficients by transposition of the Polynomial Coefficients.

In terms of the above polynomial this gives:--

$$-bx + y - a = 0 \qquad \textbf{Eqn.13}$$

in which -b's are mapped into Column Two and ones (representing the coefficient of y) into Column Three. a's are abandoned: They could be used to calculate test intercepts but such is inconvenient.

(C) Install the NE Coefficients in a three-dimensional solutions tableau noting as above that all y coefficients are unity.

(D) Compute and Install the Moduli:-

$$m_1 = \sqrt{b_2^2 + 1} \qquad \textbf{Eqn.14}$$

$$m_2 = \sqrt{b_1^2 + 1} \qquad \textbf{Eqn.15}$$

$$m_3 = \sqrt{b_3^2 + 1} \qquad \textbf{Eqn.16}$$

for both sets of line pairs.

(E) Scale the NE Coefficients by the moduli.

(F) Generate the Bisectors by Differencing of the Normal Equations.

(G) Compute and Install any Required Orthogonal Bisectors.

(H) Solve for the Goniometric Co-Ordinates by Simultaneous Solution of the Bisectors' Normal Equations.

The Structure of the Solutions Tableau is illustrated in Table One.

k=1	AB/AC			
		i=1	i=2	i=3
	j=1	m_1	A_1=-b_1*m_1	B_1=1*m_1
	j=2	m_2	A_2=-b_2*m_1	B_2=1*m_2
	j=3		u_1=A_1-A_2	v_1=B_1-B_2
k=2	AC/BC			
		i=1	i=2	i=3
	j=1	m_3	A_3=-b_2*m_3	B_3=1*m_3
	j=2	m_1	A_4=-b_3*m_1	B_4=1*m_1
	j=3		u_2=A_2-A_3	v_2=B_2-B_3

Table One
The Structure of the Goniocentric Solutions Matrix

It is quite likely that this algorithm could further be simplified to enhance clarity and precision.

Summary of Program Test Results

The Program ELONGA.BAS of Appendix One was tested by application to a 26-element Delaunay triangulation to yield shape statistics for each triangle.

This triangulation is illustrated by Figure One.

The contents of MUNSELON.DAT, represented in Appendix Two, defined the cartesian co-ordinates for the vertices of each triangle, and constituted the primary test data.

The output statistics are tabulated in Appendix Three.

Subsidiary results were computed for Perimeter (using Pythagoras Theorem) and Area (using Heron's Formula).

Because the MUNSTER test data includes a diversity of scalene, isosceles and right-angled cells it may sanguinely be expected that ELONGA.BAS will hold good for any small two-dimensional triangulation.

Manual centroid plotting and selected manual elaborations substantiated the accuracy of ELONGA.BAS with the MUNSTER test data.

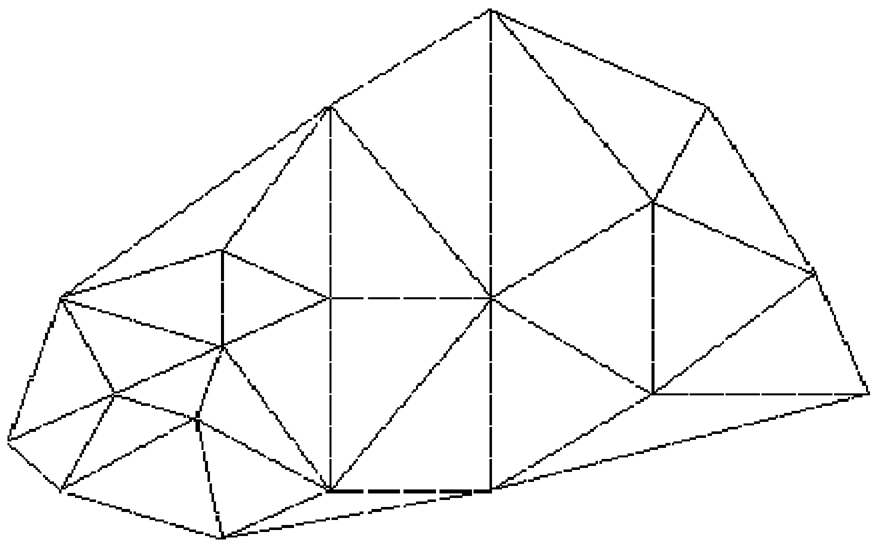

Figure One
The Delaunay Triangulation of All Munster Points

To investigate the behavior of ELONGA.BAS with extremely extended triangles a second test file was developed for the first twelve triangles defined by:-

[Name of Number, N],1,2,2,1,10^N,10^N

Results for this second test file showed that elongations were very precise for side-length ratios up to about 10^7 but computational errors introduced serious inaccuracies around 10^8 and 10^9, though accuracy was powerfully restored for 10^{10} and 10^{11} before utter failure at 10^{12}.

Notation

α_1	The Intercept of the First Bisector
α_2	The Intercept of the Second Bisector
a	an Intercept
a_1	The Side 1 Intercept
a_2	The Side 2 Intercept
a_3	The Side 3 Intercept
A	The Normal Equation Coefficient of x
A_1	The Normal Equation Coefficient of x for Side 1
A_2	The Normal Equation Coefficient of x for Side 2
A_3	The Normal Equation Coefficient of x for Side 3
β_1	The Grade of the First Bisector
β_2	The Grade of the Second Bisector
b	a Grade
b_1	The Side 1 Grade
b_2	The Side 2 Grade
b_3	The Side 3 Grade
B	The Normal Equation Coefficient of y
B_1	The Normal Equation Coefficient of y for Side 1
B_2	The Normal Equation Coefficient of y for Side 2
B_3	The Normal Equation Coefficient of y for Side 3
γ	The Arithmetised Elongation
C_1	The Normal Equation Line Constant for Side 1
C_2	The Normal Equation Line Constant for Side 2
C_3	The Normal Equation Line Constant For Side 3
δ	The Centroids' Interdistance (Separation)
η	The Triangle Elongation
l	(subscript) of Triangle Side Number l
m_1	The Real Modulus of Side 2
m_2	The Real Modulus of Side 1
m_3	The Real Modulus of Side 3
P	The Triangle Perimeter
u_k	The Difference of Grade Coefficients for Side k
v_k	The Difference of Ordinate Coefficients (i.e. the Moduli Difference) for Side k
x	an x Co-ordinate
x_v	The x Co-ordinate of the Barycenter
x_g	The x Co-ordinate of the Goniocenter
x_i	The x Co-ordinate of Vertex i
y	a y Co-ordinate
y_v	The y Co-ordinate of the Barycenter
y_g	The y Co-ordinate of the Goniocenter
y_i	The y Co-ordinate of Vertex i

APPENDIX ONE

The Program ELONGA.BAS

```
'     PROGRAM ELONGA.BAS
'     A PROGRAM TO COMPUTE THE ELONGATION OF A TRIANGLE
'     TOGETHER WITH OTHER STATISTICS
'
'     WRITTEN BY:-
'
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'       "SOUTHGATE"
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'       WALSALL
'       WS3 3HS
'       UNITED KINGDOM
'
'       19 AUGUST 2000
'
'     THIS PROGRAM IS WRITTEN IN MICROSOFT QBASIC
'
'     *** THIS PROGRAM ELONGA.BAS SUPERCEDES PROGRAM ELONG.BAS ***
'     *** WHICH WAS DATED 10 APRIL 1999                        ***
'
' VARIABLE TYPE DEFAULTS
      DEFDBL A-H, O-R, T-Z
      DEFSTR S
      DEFINT I-K, M-N
      DEFLNG L
' SEGMENT DECLARATIONS
      DECLARE SUB CDUMP (SCP, C())
      DECLARE FUNCTION DMOD (A, B)
      DECLARE FUNCTION DSTRADDLES (C1, A, C2)
      DECLARE SUB GETTER ()
      DECLARE SUB GONIOCENTER (X1, Y1, X2, Y2, X3, Y3, XG, YG)
      DECLARE SUB ROTATE (TH, XO, YO, XI, YI, XR, YR)
      DECLARE SUB STATCALC (X(), Y(), XG, YG, XB, YB, CI, A, P, EL, EG)
      DECLARE SUB STATPRINT (IA, IB, STL, STN, XG, YG, XB, YB, CI, A, P, EL,
EG)
      DECLARE SUB TABLEHEAD (STL, N)
      DECLARE SUB TABLELINE (STN, XG, YG, XB, YB, CI, A, P, EL, EG)
      DECLARE SUB VERTEXINPUT (STL, N, SN(), XN(), YN())
      DECLARE SUB VERTEXSORT (X(), Y())
' COMMON VARIABLES
      COMMON SHARED SI4, SF10.8, SF11.6
      COMMON SHARED IA, SA
      COMMON SHARED IU, IV, IW, SP, SQ, SFN, SXU, SXV, SXW
      COMMON SHARED SC, SM, SCR
' STATIC ARRAY DEFINITIONS
```

```
      DIM X(3), Y(3)
      DIM IRT(2, 2)
      DIM SN(500), XN(500, 3), YN(500, 3)
' COMMON STATAT ARRAYS
      COMMON SHARED IRT()
' DYNAMIC ARRAY DEFINITIONS
'     ( none )
' DYNAMIC ARRAY LIMIT
'     ( none )
' DEVICE ATTRIBUTIONS
      SCREEN 12: WINDOW (1, 1)-(640, 480)
' LOGICAL UNIT, EXTENSION AND PATHNAME SETTINGS
      IU = 1: IV = 2: IW = 3
      SXU = ".DAT": SXV = ".ELO": SXW = ".ELT"
      SP = "C:\QBASIC\QBFILES\"
      SQ = "CON"
' FORMAT DEFINITIONS
      SI4 = "####"
      SF10.8 = "#.########"
      SF11.6 = "####.######"
' NUMERICAL CONSTANT DEFINITIONS
'     ( none )
' STRING CONSTANT DEFINITIONS
      SC = ":": SM = ",": SCR = CHR$(13) + CHR$(10)
' TEXT VARIABLE DEFINITIONS
'     ( none )
'
' ** THE ALGORITHM **
'
      SFN = "MUNSELON"
      VERTEXINPUT STL, N, SN(), XN(), YN()
      TABLEHEAD STL, N
      OPEN "O", IV, SP + SFN + SXV
      FOR I = 1 TO N
        PRINT "TRIANGLE "; I; " OF "; N; " DONE"
        FOR J = 1 TO 3
          X(J) = XN(I, J): Y(J) = YN(I, J)
        NEXT J
        VERTEXSORT X(), Y()
        GONIOCENTER X(1), Y(1), X(2), Y(2), X(3), Y(3), XG, YG
        STATCALC X(), Y(), XG, YG, XB, YB, CI, A, P, EL, EG
        STATPRINT 30, 55, STL, SN(I), XG, YG, XB, YB, CI, A, P, EL, EG
        TABLELINE SN(I), XG, YG, XB, YB, CI, A, P, EL, EG
      NEXT I
      CLOSE
      END

      SUB CDUMP (SCP, C())
```

```
' A SUBROUTINE TO PRINT THE CONTENTS OF ARRAY C TO THE SCREEN
'     ARGUMENT:
'        SCP    THE LOCATION DESCRIPTIVE CAPTION
'        C()    THE BISECTOR SOLUTIONS MATRIX
' ( SF11.6 IS COMMON SHARED )
'
      IU = 12
      CLS : IL = LEN(SCP)
      PRINT SCP: PRINT STRING$(IL, 45): PRINT
      STI = "AB/AC"
      FOR K = 1 TO 2
        PRINT "K="; K; "     FOR "; STI: PRINT
        IT = 0
        FOR I = 1 TO 3
          IT = IT + IU
          PRINT TAB(IT); "I="; I;
        NEXT I
        PRINT
        FOR J = 1 TO 3
          IT = 0: PRINT "J="; J;
          FOR I = 1 TO 3
            IT = IT + IU
            PRINT TAB(IT);
            PRINT USING SF11.6; C(I, J, K);
          NEXT I
          PRINT
        NEXT J
        PRINT : PRINT : STI = "AC/BC"
      NEXT K
      GETTER
      END SUB

      FUNCTION DMOD (A, B)
' A FUNCTION TO RETURN THE REAL MODULUS OF A AND B
'     ARGUMENTS:
'        A      THE ADJACENT ARGUMENT
'        B      THE OPPOSITE ARGUMENT
'
      DMOD = SQR(A * A + B * B)
      END FUNCTION

      FUNCTION DSTRADDLES (C1, A, C2)
' A FUNCTION THAT RETURNS ZERO IF A LIES WITHIN THE RANGE C1 TO C2
' ( OR C2 TO C1 FOR C1<C2 ) INCLUSIVE.
' OTHERWISE DSTRADDLES RETURNS UNITY.
'     ARGUMENTS:
'        C1     THE FIRST  LIMIT
'        A      THE RANGE INCLUSION CANDIDATE NUMBER
```

```
'         C2     THE SECOND LIMIT
'
      IF C2 < C1 THEN
           TP = C1
           C1 = C2
           C2 = TP
      END IF
      IF A >= C1 AND A <= C2 THEN
           DSTRADDLES = 0
         ELSE
           DSTRADDLES = 1
      END IF
      END FUNCTION

      SUB GETTER
' A SUBROUTINE TO ACCEPT A KEYSTROKE AS SA AND TO YIELD ITS ASCII
CODE AS IA
' ( SA AND IA ARE COMMON SHARED )
      DO
         SA = INKEY$
      LOOP UNTIL SA <> ""
      IA = ASC(SA)
      END SUB

      SUB GONIOCENTER (X1, Y1, X2, Y2, X3, Y3, XG, YG)
' A SUBROUTINE TO LOCATE THE GONIOMETRIC CENTER OF THE TRIANGLE
DEFINED BY
' THE ORDERED VERTICES (X1,Y1),(X2,Y2),(X3,Y3).
'      ARGUMENTS:
'         X1     THE X CO-ORDINATE OF THE FIRST  VERTEX
'         Y1     THE Y CO-ORDINATE OF THE FIRST  VERTEX
'         X2     THE X CO-ORDINATE OF THE SECOND VERTEX
'         Y2     THE Y CO-ORDINATE OF THE SECOND VERTEX
'         X3     THE X CO-ORDINATE OF THE THIRD  VERTEX
'         Y3     THE X CO-ORDINATE OF THE THIRD  VERTEX
'         XG     THE X CO-ORDINATE OF THE GONIOMETRIC CENTER
'         YG     THE Y CO-ORDINATE OF THE GONIOMETRIC CENTER
' ( ARRAY IRT() IS COMMON SHARED)
' ** NOTE: VERTICES MUST BE SORTED IN X FOR ALL X, AND NO X OR Y MUST
BE EQUAL. ***
' **       VERTICES MAY BE PREPARED USING SUBROUTINE VERTEXSORT.
   ***
'
      ERASE IRT
      DIM A(2), B(2), XX(2), YY(2), C(3, 3, 2), CP(4, 3)
      XX(1) = X1: XX(2) = X3: YY(1) = Y1: YY(2) = Y3
' COMPUTE THE FIRST-DEGREE POLYNOMIAL COEFFICIENTS OF THE EDGES
      CP(2, 1) = (Y2 - Y1) / (X2 - X1): CP(1, 1) = Y1 - CP(2, 1) * X1
```

```
      CP(2, 2) = (Y3 - Y1) / (X3 - X1): CP(1, 2) = Y1 - CP(2, 2) * X1
      CP(2, 3) = (Y3 - Y2) / (X3 - X2): CP(1, 3) = Y2 - CP(2, 3) * X2
' COMPUTE THE NORMAL EQUATION COEFFICIENTS OF THE EDGES
      C(2, 1, 1) = -CP(2, 1): C(3, 1, 1) = 1
      C(2, 2, 1) = -CP(2, 2): C(3, 2, 1) = 1
      C(2, 1, 2) = -CP(2, 2): C(3, 1, 2) = 1
      C(2, 2, 2) = -CP(2, 3): C(3, 2, 2) = 1
' COMPUTE AND PLACE THE MODULI
      C(1, 1, 1) = SQR(C(2, 2, 1) ^ 2 + 1)
      C(1, 2, 1) = SQR(C(2, 1, 1) ^ 2 + 1)
      C(1, 1, 2) = SQR(C(2, 2, 2) ^ 2 + 1)
      C(1, 2, 2) = SQR(C(2, 1, 2) ^ 2 + 1)
' COMPUTE THE BISECTORS
'   STAGE A:- MULTIPLY THROUGH BY THE MODULII
      FOR K = 1 TO 2
        FOR J = 1 TO 2
          FOR I = 2 TO 3: C(I, J, K) = C(I, J, K) * C(1, J, K): NEXT I
        NEXT J
      NEXT K
'   STAGE B:- DIFFERENCE THE FIRST AND SECOND EQUATION COEFFICENTS
      FOR K = 1 TO 2
        FOR I = 2 TO 3: C(I, 3, K) = C(I, 1, K) - C(I, 2, K): NEXT I
      NEXT K
'   STAGE C:- COMPUTE BISECTOR GRADIENTS AND INTERCEPTS
      B(1) = C(3, 3, 1) / C(2, 3, 1): A(1) = YY(1) - B(1) * XX(1)
      B(2) = C(3, 3, 2) / C(2, 3, 2): A(2) = YY(2) - B(2) * XX(2)
' FIND ANY ORTHOGONAL BISECTORS REQUIRED
      IF DSTRADDLES(CP(1, 1), A(1), CP(1, 2)) THEN
          IRT(1, 2) = 1: B(1) = -1 / B(1)
      END IF
      IF DSTRADDLES(CP(1, 1), A(2), CP(1, 3)) THEN
          IRT(2, 2) = 1: B(2) = -1 / B(2)
      END IF
' SOLVE THE SIMULTANEOUS EQUATION FOR XG,YG
      XG = (B(1) * X1 - B(2) * X3 - Y1 + Y3) / (B(1) - B(2))
      YG = B(1) * (XG - X1) + Y1
' TERMINATE
      END SUB

      SUB ROTATE (TH, XO, YO, XI, YI, XR, YR)
' A SUBROUTINE TO ROTATE A PAIR OF CARTESIAN CO-ORDINATES (X,Y)
' DEXTRALLY ABOUT XO,YO THROUGH ANGLE TH
'     ARGUMENTS:
'       TH    THE ANGLE OF ROTATION ( RADIANS )
'       XO    THE ABSCISSAL CENTER OF ROTATION
'       YO    THE ORDINAL   CENTER OF ROTATION
'       XI    THE UNROTATED X CO-ORDINATE
'       YI    THE UNROTATED Y CO-ORDINATE
```

```
'        XR     THE ROTATED   X CO-ORDINATE
'        YR     THE ROTATED   Y CO-ORDINATE
'
      AC = COS(TH): AI = SIN(TH): X = XI - XO: Y = YI - YO
      XR = XO + X * AC + Y * AI
      YR = YO + Y * AC - X * AI
      END SUB

      SUB STATCALC (X(), Y(), XG, YG, XB, YB, CI, A, P, EL, EG)
' A SUBROUTINE TO CALCULATE TRIANGLE STATISTICS
'     ARGUMENTS:
'        X()    A TRIANGLE VERTEX X CO-ORDINATE
'        Y()    A TRIANGLE VERTEX Y CO-ORDINATE
'        XG     THE GONIOMETRIC CENTROID X CO-ORDINATE
'        YG     THE GONIOMETRIC CENTROID Y CO-ORDINATE
'        XB     THE BARYMETRIC  CENTROID X CO-ORDINATE
'        YB     THE BARYMETRIC  CENTROID Y CO-ORDINATE
'        CI     THE CENTROIDS' SEPARATION
'        A      THE TRIANGLE AREA
'        P      THE TRIANGLE PERIMETER
'        EL     THE TRIANGLE ELONGATION
'        EG     THE ARITHMETISED ELONGATION
'
      DIM D(3)
      XT = 0#: YT = 0#
      FOR I = 1 TO 3: XT = XT + X(I): YT = YT + Y(I): NEXT I
      XB = XT / 3: YB = YT / 3
      CI = DMOD(XB - XG, YB - YG)
      D(1) = DMOD(X(2) - X(1), Y(2) - Y(1))
      D(2) = DMOD(X(3) - X(2), Y(3) - Y(2))
      D(3) = DMOD(X(1) - X(3), Y(1) - Y(3))
      P = 0#: FOR I = 1 TO 3: P = P + D(I): NEXT I
      PS = P / 2: PR = PS
      FOR I = 1 TO 3: PR = PR * (PS - D(I)): NEXT I
      A = SQR(PR)
      EL = 6 * CI / P
      EG = 2 / -LOG(EL)
      END SUB

      SUB STATPRINT (IA, IB, STL, STN, XG, YG, XB, YB, CI, A, P, EL, EG)
' A SUBROUTINE TO PRINT TRIANGLE STATISTICS TO FileName.ELO
'     ARGUMENTS:
'        IA     THE FIRST  TABULATION POSITION
'        IB     THE SECOND TABULATION POSITION
'        STL    THE BATCH TITLE
'        STN    THE TRIANGLE NAME
'        XG     THE GONIOMETRIC CENTROID X CO-ORDINATE
'        YG     THE GONIOMETRIC CENTROID Y CO-ORDINATE
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```
'        XB     THE BARYMETRIC  CENTROID X CO-ORDINATE
'        YB     THE BARYMETRIC  CENTROID Y CO-ORDINATE
'        CI     THE CENTROIDS' SEPARATION
'        A      THE TRIANGLE AREA
'        P      THE TRIANGLE PERIMETER
'        EL     THE TRIANGLE ELONGATION
'        EG     THE ARITHMETISED ELONGATION
' ( ARRAY IRT IS COMMON SHARED )
' ( IV, SP, SFN AND SXV ARE COMMON SHARED )
'
      PRINT #IV, "TRIANGLE STATISTICS": PRINT #IV, STRING$(19, 45): PRINT #IV,
      PRINT #IV, "BATCH NAME:    "; TAB(IA); STL
      PRINT #IV, "THIS TRIANGLE: "; TAB(IA); STN
      PRINT #IV,
      PRINT #IV, TAB(IA); "X"; TAB(IB); "Y"
      PRINT #IV, "GONIOMETRIC CENTROID"; TAB(IA); XG; TAB(IB); YG
      PRINT #IV, "BARYMETRIC  CENTROID"; TAB(IA); XB; TAB(IB); YB
      PRINT #IV,
      PRINT #IV, "CENTROIDS' SEPARATION"; TAB(IA); CI
      PRINT #IV, "AREA"; TAB(IA); A
      PRINT #IV, "PERIMETER"; TAB(IA); P
      PRINT #IV, "ELONGATION"; TAB(IA); EL
      PRINT #IV, "ARITHMETISED ELONGATION"; TAB(IA); EG
      PRINT #IV,
      IF IRT(1, 1) = 1 THEN PRINT #IV, "FIRST  BISECTOR ABSCISSAL ANOMALLY
DETECTED"
      IF IRT(2, 1) = 1 THEN PRINT #IV, "SECOND BISECTOR ABSCISSAL
ANOMALLY DETECTED"
      IF IRT(1, 2) = 1 THEN PRINT #IV, "FIRST  ORTHOGONAL AVAILED"
      IF IRT(2, 2) = 1 THEN PRINT #IV, "SECOND ORTHOGONAL AVAILED"
      PRINT #IV,
      PRINT #IV,
      END SUB

      SUB TABLEHEAD (STL, N)
' A SUBROUTINE TO EMPLACE THE OUTPUT TABLE HEADING FOR TRIANGLE
STATISTICS
'     ARGUMENTS:
'        STL    THE BATCH TITLE
'        N      THE NUMBER OF TRIANGLES
' ( IW, SP, SFN, SXW AND SM ARE COMMON SHARED )
'
      OPEN "O", IW, SP + SFN + SXW
      CLOSE IW
      OPEN "A", IW, SP + SFN + SXW
      PRINT #IW, "BATCH NAME:"; SM; STL
      PRINT #IW, "NUMBER OF TRIANGLES:"; SM;
      PRINT #IW, USING SI4; N
```

```
      PRINT #IW,
      PRINT #IW, "TRIANGLE"; SM; "CENTROIDS'"; SM; SM; SM; SM; "CENTROIDS'";
SM;
      PRINT #IW, "AREA"; SM; "PERIMETER"; SM; "ELONGATION"; SM;
"ARITHMETISED"
      PRINT #IW, "NAME"; SM; "GONIOMETRIC"; SM; SM; "BARYMETRIC"; SM; SM;
"SEPARATION"; SM; SM; SM; SM; "ELONGATION"
      PRINT #IW, SM; "X"; SM; "Y"; SM; "X"; SM; "Y"
      PRINT #IW,
      END SUB

      SUB TABLELINE (STN, XG, YG, XB, YB, CI, A, P, EL, EG)
' A SUBROUTINE TO APPEND TRIANGLE STATISTICS TO THE TABLE FILE
FileName.ELT
'     ARGUMENTS:
'        STN    THE TRIANGLE NAME
'        XG     THE GONIOMETRIC CENTROID X CO-ORDINATE
'        YG     THE GONIOMETRIC CENTROID Y CO-ORDINATE
'        XB     THE BARYMETRIC  CENTROID X CO-ORDINATE
'        YB     THE BARYMETRIC  CENTROID Y CO-ORDINATE
'        CI     THE CENTROIDS' SEPARATION
'        A      THE TRIANGLE AREA
'        P      THE TRIANGLE PERIMETER
'        EL     THE TRIANGLE ELONGATION
'        EG     THE ARITHMETISED ELONGATION
' ( IW AND SM ARE COMMON SHARED )
'
      PRINT #IW, STN; SM;
      PRINT #IW, USING SF10.8; XG;
      PRINT #IW, SM;
      PRINT #IW, USING SF10.8; YG;
      PRINT #IW, SM;
      PRINT #IW, USING SF10.8; XB;
      PRINT #IW, SM;
      PRINT #IW, USING SF10.8; YB;
      PRINT #IW, SM;
      PRINT #IW, USING SF10.8; CI;
      PRINT #IW, SM;
      PRINT #IW, USING SF10.8; A;
      PRINT #IW, SM;
      PRINT #IW, USING SF10.8; P;
      PRINT #IW, SM;
      PRINT #IW, USING SF10.8; EL;
      PRINT #IW, SM;
      PRINT #IW, USING SF10.8; EG
      END SUB

      SUB VERTEXINPUT (STL, N, SN(), XN(), YN())
```

```
' A SUBROUTINE TO COLLECT N SETS OF TRIANGLE VERTICES
' FROM THE FILE ELONG.DAT
'     ARGUMENTS:
'        STL    THE BATCH TITLE
'        N      THE NUMBER OF TRIANGLES
'        SN()   THE ARRAY OF TRIANGLE NAMES
'        XN()   A TRIANGLE VERTEX X CO-ORDINATE ( FOR A NAMED TRIANGLE )
'        YN()   A TRIANGLE VERTEX Y CO-ORDINATE ( FOR A NAMED TRIANGLE )
' ( IU, SFN, SXU, AND SP ARE COMMON SHARED )
'
      OPEN "I", IU, SP + SFN + SXU
      INPUT #IU, STL
      INPUT #IU, N
      FOR I = 1 TO N
        INPUT #IU, SN(I), XN(I, 1), YN(I, 1), XN(I, 2), YN(I, 2), XN(I, 3), YN(I, 3)
      NEXT I
      CLOSE IU
      END SUB

      SUB VERTEXSORT (X(), Y())
' A SUBROUTINE TO (BUBBLE) SORT THE TRIANGLE VERTICES.
' SHOULD IT BE DETECTED THAT ANY X CO-ORDINATES ARE EQUIVALENT OR
' THAT ANY Y CO-ORDINATES ARE EQUIVALENT THEN A RIGID ROTATION IS
APPLIED
' TO THE SECOND AND THIRD INPUT POINTS ABOUT THE FIRST USING A
SMALL
' ARBITRARY ANGLE. IT IS ASSUMED THAT NO EQUIVALENCIES WOULD
SURVIVE THIS
' PROCEDURE.
'     ARGUMENTS:
'        X()    A TRIANGLE VERTEX X CO-ORDINATE
'        Y()    A TRIANGLE VERTEX Y CO-ORDINATE
'
      IDT = 0: DA = .000000000001#
      IF X(1) = X(2) OR Y(1) = Y(2) THEN IDT = 1
      IF X(1) = X(3) OR Y(1) = Y(3) THEN IDT = 1
      IF X(2) = X(3) OR Y(2) = Y(3) THEN IDT = 1
      IF IDT = 1 THEN
        FOR I = 1 TO 2
          ROTATE DA, X(1), Y(1), X(I + 1), Y(I + 1), XR, YR
          X(I + 1) = XR: Y(I + 1) = YR
        NEXT I
      END IF
      FOR I = 1 TO 2
        FOR J = 1 TO 2
          IF X(J) > X(J + 1) THEN
            XT = X(J): YT = Y(J)
            X(J) = X(J + 1): Y(J) = Y(J + 1)
```

```
        X(J + 1) = XT: Y(J + 1) = YT
      END IF
   NEXT J
NEXT I
END SUB
```

APPENDIX TWO

MUNSIN.CAL
A Table of MUNSTER Test Data

THE DELAUNAY TRIANGLES FOR ALL MUNSTER POINTS

26

T1	0.55	0.15	0.9	0.25	0.7	0.25
T2	0.55	0.15	0.4	0.15	0.3	0.1
T3	0.4	0.35	0.3	0.3	0.4	0.15
T4	0.2	0.25	0.3	0.3	0.15	0.35
T5	0.2	0.25	0.15	0.35	0.1	0.2
T6	0.55	0.15	0.55	0.35	0.4	0.15
T7	0.55	0.65	0.55	0.35	0.7	0.45
T8	0.15	0.15	0.3	0.1	0.275	0.225
T9	0.275	0.225	0.4	0.15	0.3	0.3
T10	0.3	0.1	0.4	0.15	0.275	0.225
T11	0.2	0.25	0.15	0.15	0.275	0.225
T12	0.3	0.4	0.3	0.3	0.4	0.35
T13	0.2	0.25	0.1	0.2	0.15	0.15
T14	0.2	0.25	0.275	0.225	0.3	0.3
T15	0.9	0.25	0.85	0.375	0.7	0.25
T16	0.55	0.65	0.7	0.45	0.75	0.55
T17	0.55	0.15	0.7	0.25	0.55	0.35
T18	0.55	0.35	0.4	0.35	0.4	0.15
T19	0.7	0.45	0.85	0.375	0.75	0.55
T20	0.4	0.55	0.55	0.35	0.55	0.65
T21	0.7	0.45	0.55	0.35	0.7	0.25
T22	0.7	0.45	0.7	0.25	0.85	0.375
T23	0.4	0.55	0.3	0.4	0.4	0.35
T24	0.3	0.4	0.15	0.35	0.3	0.3
T25	0.4	0.55	0.4	0.35	0.55	0.35
T26	0.4	0.55	0.15	0.35	0.3	0.4

APPENDIX THREE

MUNSOUT.CAL
A Table of MUNSTER Triangle Statistics

BATCH NAME: THE DELAUNAY TRIANGLES FOR ALL MUNSTER POINTS
NUMBER OF TRIANGLES: 26

TRIANGLE NAME	CENTROIDS' GONIOMETRIC X	Y	BARYMETRIC X	Y	CENTROIDS' SEPARATION	AREA	PERIMETER	ELONGATION	ARITHMETISED ELONGATION
T1	0.70813603	0.2231285	0.71666667	0.21666667	0.01070173	0.01	0.74428306	0.08627145	0.81624104
T2	0.40342621	0.13548633	0.41666667	0.13333333	0.01341436	0.00375	0.51675437	0.15575323	1.07556813
T3	0.35935628	0.28423708	0.36666667	0.26666667	0.01903053	0.01	0.49208096	0.2320415	1.36907623
T4	0.21464466	0.29393398	0.21666667	0.3	0.00639414	0.00625	0.38172068	0.10050506	0.87049355
T5	0.15606602	0.26464466	0.15	0.26666667	0.00639414	0.00625	0.38172068	0.10050506	0.87049355
T6	0.5	0.2	0.5	0.21666667	0.01666667	0.015	0.6	0.16666667	1.11622125
T7	0.61162041	0.46513878	0.6	0.48333333	0.02158878	0.0225	0.73027756	0.17737461	1.15640936
T8	0.24650871	0.16059398	0.24166667	0.15833333	0.00534378	0.00875	0.43136317	0.07432868	0.76945023
T9	0.30838974	0.23735167	0.325	0.225	0.02069938	0.005625	0.4051083	0.30657551	1.69163062
T10	0.32584702	0.15284783	0.325	0.15833333	0.00555051	0.006875	0.38505268	0.08648964	0.81708336
T11	0.21316684	0.21821244	0.20833333	0.20833333	0.01099816	0.004375	0.33663414	0.1960257	1.2273632
T12	0.3309017	0.35	0.33333333	0.35	0.00243163	0.005	0.3236068	0.04508497	0.64532651
T13	0.14301898	0.19301898	0.15	0.2	0.00987265	0.00375	0.29431748	0.20126537	1.24755868
T14	0.26035534	0.25428932	0.25833333	0.25833333	0.00452134	0.003125	0.26991728	0.10050506	0.87049355
T15	0.83031356	0.29718002	0.81666667	0.29166667	0.01471852	0.0125	0.52988536	0.16666077	1.11619921
T16	0.6927051	0.5309017	0.66666667	0.55	0.03229156	0.0125	0.5854102	0.33096344	1.80873144
T17	0.60351838	0.25	0.6	0.25	0.00351838	0.015	0.56055513	0.03765955	0.60991069
T18	0.45	0.3	0.45	0.28333333	0.01666667	0.015	0.6	0.16666667	1.11622125
T19	0.75229183	0.46743061	0.76666667	0.45833333	0.01701166	0.009375	0.48106494	0.21217496	1.29003623
T20	0.48837959	0.53486122	0.5	0.51666667	0.02158878	0.0225	0.73027756	0.17737461	1.15640936
T21	0.64648162	0.35	0.65	0.35	0.00351838	0.015	0.56055513	0.03765955	0.60991069
T22	0.75328963	0.36377557	0.75	0.35833333	0.00635921	0.015	0.56296134	0.06777603	0.74306719
T23	0.35935628	0.41576292	0.36666667	0.43333333	0.01903053	0.01	0.49208096	0.2320415	1.36907623
T24	0.26396204	0.35	0.25	0.35	0.01396204	0.0075	0.41622777	0.20126537	1.24755868
T25	0.45	0.4	0.45	0.41666667	0.01666667	0.015	0.6	0.16666667	1.11622125
T26	0.28294695	0.42232671	0.28333333	0.43333333	0.0110134	0.00875	0.65854766	0.10034263	0.86988115