The Intervisibility of Eminences Upon a Sphere

by
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This concerns the geometric criteria of rectilineal intervisibility between two points raised upon a planetary surface.

The planet is assumed a perfect sphere. The Earth is of course an oblate spheroid but in the ensuing simplified development is treated as a sphere.

All optical interactions with the atmosphere or other matter are disconsidered, including such phenomena as refraction, diffraction and its expressions as mirage.

In this study a Mean Terrestrial Radius of 6370949 meters is assumed.

The Point Interangle, γ

The Point Interangle, γ, is the angle at the planetary center between the local radii of the two eminences A and B. Such a local radius is the sum of the planetary radius and the height of the eminence above some nominal datum, e.g. mean sea level. Whilst in this work an eminence is usually a hill summit there is no reason in principle why it should not be a feature at some intermediate elevation.

The Point Interangle may be computed from knowledge of the Latitudinal Interangle, β, and the Longitudinal Interangle, α, as well as the two separate Latitudes, ϕ_1 and ϕ_2. Conventionally, $\alpha \equiv \delta\lambda$ (change in longitude) and $\beta \equiv \delta\phi$ (change in latitude).

The location of eminences is usually sourced as sexagesimal coordinates in degrees, minutes and seconds of arc. To render such readings to interangles in radians we apply these formulae:-

$$\alpha \equiv \delta\lambda = \frac{\pi}{180}\left|-\left(\lambda D_B + \frac{\lambda M_B}{60} + \frac{\lambda S_B}{3600}\right)+\left(\lambda D_A + \frac{\lambda M_A}{60} + \frac{\lambda S_A}{3600}\right)\right|$$

Equation 1

and:-

$$\beta \equiv \delta\phi = \frac{\pi}{180}\left|\left(\phi D_B + \frac{\phi_B}{60} + \frac{\phi S_B}{3600}\right)-\left(\phi D_A + \frac{\phi M_A}{60} + \frac{\phi S_A}{3600}\right)\right|$$

Equation 2

These conversions are adapted to the signage of Northern Hemisphere coordinates of points West of the Greenwich Meridian.

Figure One illustrates the spatial relations of α, β and γ and the arcs they subtend at the planetary surface:-

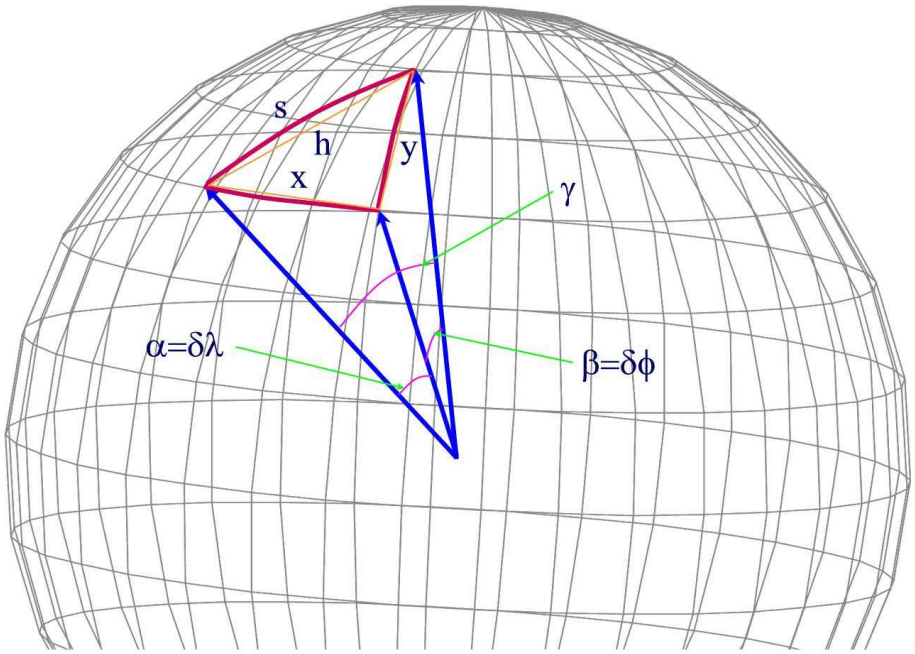

Figure One
Planet Center Interangles and their Subtended Arcs and Chords

The Haversine Equation recommended by Sinnet and Chamberlain[1,2] computes The Point Interangle, χ, as:-

$$\chi \equiv \gamma = 2\,\mathrm{Asin}\left[\sqrt{\mathrm{Sin}\left(\frac{\delta\phi}{2}\right)^2 + \mathrm{Cos}(\phi_1).\mathrm{Cos}(\phi_2).\mathrm{Sin}\left(\frac{\delta\lambda}{2}\right)^2}\right]$$

Equation 3

It is now possible to calculate a Datum Surface Arc Distance, s, between the two eminences A and B as:-

$$s = r\gamma$$

Equation 4

The Line-of-Sight Distance, T

The Line-of-Sight is a straight line subtending γ that connects the eminent points A and B. In case the line-of-sight does not intersect the planet then A and B are in principle intervisible. (Barring cloud, interposed hills, or other obstructions). If the line-of-sight dives through the ground the eminences are not intervisible.

There is a closely-associated criterion of intervisibility that we shall call the Glance Height, h_p. The Glance Height is the closest approach of the line-of-sight to the ground. The local radius at the glance height is normal to the line-of-sight, and the line-of-sight is parallel to the planetary surface where it meets the glance height.

If Glance Height, h_p, is positive the two eminences are in principle intervisible. If Glance Height is negative the bulk of the planet permanently occludes each place from the other. It should be emphasised that a positive glance height does not guarantee actual intervisibility: Often a “strategically-placed” hill or bluff frustrates perception.

Figure Two summarises the geometry:-

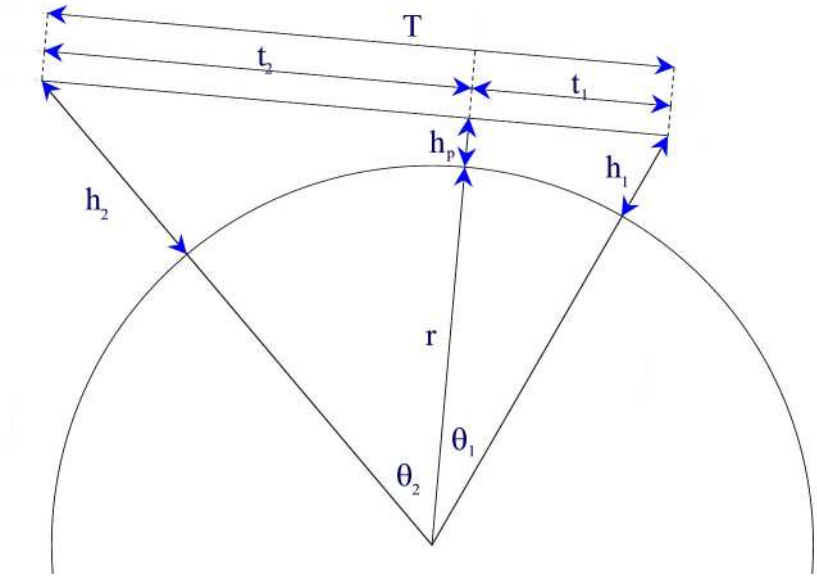

Figure Two
The Line-of-Sight and The Glance Height

Note that h_1 is The Elevation of the First Eminence and h_2 The Elevation of the Second Eminence.

The perpendicular glance height's radius bisects γ into angles θ_1 and θ_2 which are respectively subtended by the line segments t_1 and t_2 that together form the Line-of-Sight with length T.

Therefore:-

$$\gamma = \theta_1 + \theta_2$$

Equation 5

and:-

$$T = t_1 + t_2$$

Equation 6

Now by Pythagoras:-

$$(r + h_1)^2 = (r + h_p)^2 + t_1^{\,2}$$

Equation 7

and:-

$$(r + h_2)^2 = (r + h_p)^2 + t_2^{\,2}$$

Equation 8

Re-arranging and equivalating the square of the glance height radius we obtain:-

$$(r + h_1)^2 - t_1^{\,2} = (r + h_2)^2 - t_2^{\,2}$$

Equation 9

Therefore:-

$$(r+h_1)^2-(r+h_2)^2={t_1}^2-{t_2}^2=(t_1+t_2)(t_1-t_2)=T(t_1-t_2)$$

Equation 10

Now allow that by definition:-

$$U=t_1-t_2$$

Equation 11

Accordingly:-

$$(r+h_1)^2-(r+h_2)^2=TU$$

Equation 12

Or:-

$$U=\frac{(r+h_1)^2-(r+h_2)^2}{T}$$

Equation 13

From the foregoing we may observe that:-

$$t_1=\frac{(t_1+t_2)+(t_1-t_2)}{2}=\frac{T+U}{2}$$

Equation 14

It is now apparent that knowledge of T will permit the computation of t_1 from which an appeal to Pythagoras will lead directly to a signed value of the glance height.

Now by The Cosine Rule:-

$$T=\sqrt{(r+h_1)^2+(r+h_2)^2-2(r+h_1)(r+h_2)Cos\gamma}$$

Equation 15

whilst:-

$$(r+h_1)^2={t_1}^2+(r+h_p)$$

Equation 16

(Alternatively, h_2 and t_2 may be used instead of h_1 and t_1).

Re-arrangement of Equation Sixteen enables us to identify the Glance Height as:-

$$h_p = \sqrt{(r + h_1)^2 - t_1^{\,2}} - r$$

Equation 17

Computational Outputs

Using both MATHCAD® (a mathematician's scratchpad utility) and the spreadsheet MicroSoft EXCEL® I was able to implement the foregoing trigonometry to compute the crow-flight separations and the glance heights between several British and Irish eminences.

Appendix One presents [INTERVIS.xls].C, a tabulation of locational statistics for twenty British and Irish Hills and isles. Appendix Two [[INTERVIS.xls].HP tabulates the four hundred intervisibility glance heights computed amongst them. Geometric intervisibility is reciprocal so clearly only 190 of the tabulated intervisibilities are non-redundant.

Some Applications of The Glance Height Method

Eminence intervisibility has a host of obvious serious uses in microwave telegraphy, navigation and surveying but it also has a number of interesting implications for hill walkers, folklorists and antiquarians.

Harboro Rocks[a]

On the morning of 11 July 2000 I walked through Carsington Pasture to The Harboro Rocks (variously spelt "Harborough" etc.). The Harboro Rocks is a limestone crag on The White Peak. It was a pleasant day of sunshine and cloud with a brisk breeze. When I gained the trigonometrical point I paused for a picnic lunch and gazed South over the Mercian Plateau. The air was unusually clear for the English Midlands and I was astonished to descry the unmistakable profile of The Malvern Hills very far to the South. As I continued to view I discerned the almost equally characteristic profile of The Wrekin, nearer and more distinct but still very distant. I was lucky that neither sun-glare nor haze hid these hills.

In the case of the Harboro Rocks and The Worcestershire Beacon (representing The Malvern Hills), the Ground Distance, d, proved to be 119.782 kms (some 74.4291 miles) and the Glance Height +120 meters, proving simple intervisibility. The Ground Distance from the Harboro Rocks to The Wrekin proves to be 77.330 kms (48.0506 miles) with a glance height of +275 meters.

The respective line-of-sight T distances were 1119.788 kms (74.4328 miles) and 77.334 kms (48.0531 miles).

Figure Three
The Harboro (or Harborough) Rocks
Coordinates: 53°5'40"N:1°38'20"W
Elevation: 379 meters
Date: pm 11 July 2000

Merrick

Merrick (Kirkcudbrightshire) is, at some 843 meters, the culmination of Scotland's Southern Uplands.

Like Snowdon (Caernarvonshire, North Wales) and Snaefell (The Isle of Man) its isolated eminence lends it spectacular panoramas from which, it is said, distant and surprising hills are visible.

Our calculations show that Schiehallion[b] (Perthshire), Goat Fell[c] (Arran), Cairnpapple Hill (West Lothian) and Snaefell all show large positive glance heights and are easily intervisible. Snowdon is, however, too remote and any alleged sightings of either mountain from the other must depend upon atmospheric refraction.

Snowdon

Snowdon, in Caernarfonshire, North Wales is the highest mountain in England and Wales: But it is shorter than several Scottish hills and only two meters higher than Schiehallion.

The Wrekin (Shropshire), Cader Idris (Merionethshire, Mid-Wales) and Snaefell are all intervisible, as are the Wicklow Mountains (represented by Lugnaquillia Mountain).

Figure Four
Snowdon
Coordinates: 53°4’7”N:4°4’34”W
Elevation: 1085 meters
Date: pm 13 August 1996

Snaefell

The highest summit of The Isle of Man is an excellent vantage and Goat Fell, Cader Idris, Snowdon and Merrick all have large positive glance heights with regard to it.

In practice, Cader Idris is probably occluded by the bulk of Snowdonia.

Cairnpapple Hill

It is sometimes asserted that Goat Fell, Schiehallion and The Bass Rock off the East Lothian Coast can all be seen from Cairnpapple Hill near Bathgate. But it is the case that there is at least one higher hill in the immediate vicinity of Cairnpapple.

Calculations demonstrate that all three heights are geometrically visible.

Though Cairnpapple Hill and Schiehallion are geometrically intervisible one might expect them to be blocked by the bulk of The Ochils and in particular Ben Cleuch (720 meters) and its neighbors.

Iona[d]

Iona (Argyllshire) is a small island in The Inner Hebrides of Western Scotland.

It is said that it was the first landfall in Scotland from whose highest point St Columba could no longer descry his homeland, and accordingly he settled.

Slieve Snaught (Malin, Donegal) and Rathlin Island (Antrim) are reasonably proximal parts of Ireland, though the latter may well be hidden by Islay.

The respective glance heights from Dun I, Iona, are –72 meters and –148, and though theoretically intervisible with Schiehallion and Goat Fell the island is hidden from both. Such occlusions may of themselves have conduced to Iona's settlement in a lawless time.

Figure Five
Iona and Dun I
Coordinates: 56°20'29"N:6°23'48"W
Elevation: 100 meters
Date: pm 27 July 2006

Ranges to the Horizon

In the special case of the line-of-sight to the horizon, the line-of-sight at the horizon is normal to the planetary radius as it just grazes the planetary surface.

This condition is illustrated in Figure Six:-

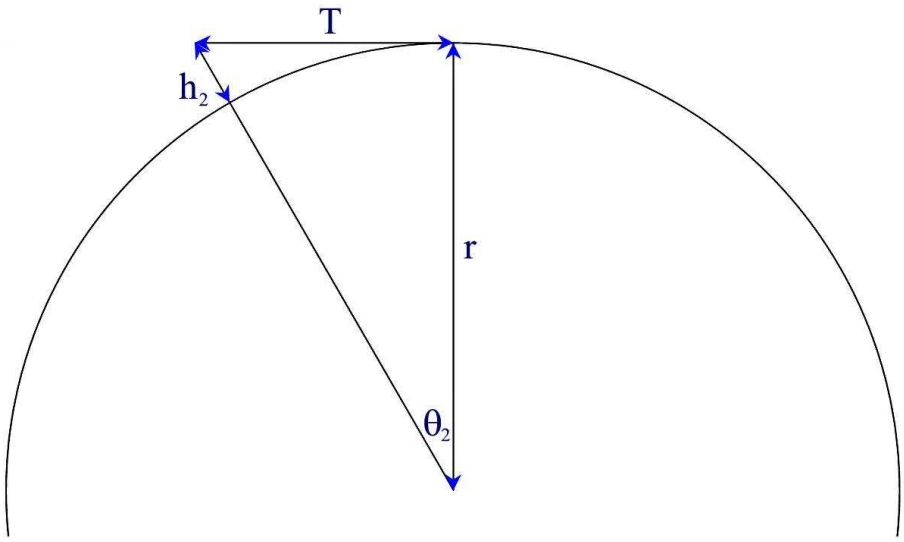

Figure Six
Distances to the Horizon

In this situation θ_1 is zero; t_1 is zero and t_2=T.
From Pythagoras we may write:-

$$(r+h_2)^2 = T^2 + r^2$$

Equation 18

or:-

$$\begin{aligned} T^2 &= (r+h_2)^2 - r^2 \\ &= r^2 + 2rt_2 + t_2{}^2 - r^2 \\ &= 2rt_2 + t_2{}^2 \\ &= t_2(2r+t_2) \end{aligned}$$

Equation 19

from which:-

$$T = \sqrt{t_2(2r+t_2)}$$

Equation 20

Arc Distance

The Arc Distance, s, is given by:-

$$s = r.\theta_2$$

Equation 21

where (in radians):-

$$\theta_2 = \text{Acos}\left[\frac{r}{r + t_2}\right]$$

Equation 22

Appendix Three gives Derivative Variables for the Twenty British and Irish Eminences, and includes Distance to the Horizon data.

Examples

In the grounds of the delightful Durleston Country Park in Dorset stands Durleston Castle, a Victorian pleasure pavilion. The pavilion and its policies are decorated with interesting stone objects that embody various astronomical and geodetic data.

In the Spring of 1994 my Wife Jana and I visited Durleston and the tablet below met our eyes:-

Figure Seven
The Convexity of the Ocean
Durleston Country Park, Swanage, Dorset
Date: pm 30 March 1994

The inscription asserts, reciprocally to our sense, that we can see one mile from an elevation of eight inches; five miles from 16 feet 7 inches; and ten miles from 66 feet 4 inches. Note that these measures are in the Imperial System of the old British Empire.

The given figures are tabulated in Appendix Four with their SI equivalents; T ands s values respectively by Equations Twenty and Twenty-One; and T and s discrepancy measures as Percentage Specific Defects, d_s, in terms of:-

$$d_s = 100\left(\frac{\delta_{eqn} - \delta_{given}}{\delta_{given}}\right)$$

Equation 23

As you can see, the maximum Percentage Specific Defect is less than minus 0.27%, which suggests that the author of the tablet employed our Pythagorean method on a spherical Earth. He would not have used a modern Mean Terrestrial Radius value and this may account for some of the discrepancies.

Alternate Criteria of Intervisibility

Reference to Appendix Three shall detect The Line-of-Sight Distance to the Sea-Level Horizon, D_T, and the very slightly different Ground Arc Distance to the Sea-Level Horizon, D_s.

The Ground Arc Distance is of course a simplification since the actual shortest path on what is really an oblate spheroid is a geodesic trace that curves transversely as well as arcing with the radius of the planet.

It can be demonstrated that eminent summits A and B are intervisible so long as:-

$$D_{T(A)} + D_{T(B)} > T$$

Equation 24

or in terms of the spherical simplification:-

$$D_{s(A)} + D_{s(B)} > s$$

Equation 25

The Line-of-Sight Horizon Distance, D_T, is given by:-

$$D_T = \sqrt{h(2r + h)}$$

Equation 26

where h is the Elevation.

Ground Horizon Distance, D_s, is computed by:-

$$D_s = r.\mathrm{Acos}\left(\frac{r}{r + h}\right)$$

Equation 27

By substitution of Equation Twenty-Six into Inequality Twenty-Four we may write:-

$$\sqrt{h_1(2r+h_1)}+\sqrt{h_2(2r+h_2)}>\sqrt{(r+h_1)^2+(r+h_2)^2-2(r+h_1)(r+h_2)Cos\gamma}$$

Equation 28

These alternate methodologies suffer, however, from the disadvantages of yielding lesser amounts of useful information than the Glance Height Method.

Notation

α	The Longitudinal Interangle (radians)
a	The Longitudinal Chord Length
A	The First Eminence (point)
β	The Latitudinal Interangle (radians)
b	The Latitudinal Chord Length
B	The Second Eminence (point)
γ	The Point Interangle (radians)
δ	A Distance
$\delta\phi$	The Change in Latitude (radians)
$\delta\lambda$	The Change in Longitude (radians)
δ_{eqn}	A Distance Computed by an Equation
δ_{given}	A Distance Quoted by a Source
d_s	The Percentage Specific Defect
$D_{\delta(E)}$	The Distance of Type δ attaching to Eminence E
D_s	The Ground Arc Distance to the Sea-Level Horizon
D_T	The Line-of-Sight Distance to the Sea-Level Horizon
ϕ_1	The Lower Latitude (Point A)
ϕ_2	The Higher Latitude (Point B)
ϕD_X	The Degrees of Latitude at Point X
ϕM_X	The Minutes of Latitude at Point X
ϕS_X	The Seconds of Latitude at Point X
h	The Elevation
h	The Interpoint Chord Length
h_1	The Elevation of the First Eminence
h_2	The Elevation of the Second Eminence
h_p	The Glance Height
λD_X	The Degrees of Latitude at Point X
λM_X	The Minutes of Latitude at Point X
λS_X	The Seconds of Latitude at Point X
π	The Ludolphine Constant
θ_1	The Angle subtending t_1
θ_2	The Angle subtending t_2
r	The Mean Terrestrial Radius
s	The Datum Surface Arc Distance
t_1	The First Segment of the Line-of-Sight
t_2	The Second Segment of the Line-of-Sight
T	The Line-of-Sight Distance
U	The Difference of Line-of-Sight Segments
x	The Longitudinal Chord subtending α

y The Latitudinal Chord subtending β

Notes

a Harboro Rocks

These are weathered tors of Dinantian Limestone amidst sheep pasture and numerous disused lead mines. Small silica brick factories maintain a somnolent trade nearby. The twentieth-century red-brick wall at 0.55:0.25 partly seals the entrance to a natural cave in which, in his tour of 1731, Daniel Defoe found a happy peasant family in residence. The lady, a miner's wife with five children, he introduces thus[7]:-

"…in a natural opening in the rock, wherein her husband had been born. The chamber within was divided by a curtain, had shelves with earthenware, pewter and brass. A hole in the roof served as a chimney. She had a few pigs and a cow enclosed outside. She earned, when she could, a few pence per day washing ore".

The Defoe party gave her half-a-crown with which she was "highly delighted". (This sum is thirty old pence or 12½ New Pence Sterling).

b Schiehallion[3,4,5,6]

"Perthshire afforded us a remarkable hill, nearly in the centre of Scotland, of sufficient height, tolerably detached from other hills, and considerably larger from east to west than from north to south, called by the people of the low country Maiden-Pap, but by the neighbouring inhabitants Schiehallion, which I have since been informed signifies in the Erse language Constant Storm;"

Thus a giant of The Enlightenment introduces another lonely eminence whose weight he will throw into the balance against a planet and the Copley Medal.

The quotation is from the Reverend Nevil Maskelyne, Fellow of the Royal Society and Astronomer-Royal to George II. As he remarked, Schiehallion is virtually in the geographic center of Scotland. It is supposed to have been venerated by the prehistoric peoples who lived there, and the name Schiehallion is said, contrary to his belief, to have a supernatural etymology. At any rate, Schiehallion is one of Scotland's more serene peaks, being in rain shadows on all quarters.

Schiehallion is a high, isolated, symmetric peak of quartzite that "afforded" Maskelyne and Hutton the mass by which, in the autumn of 1774, they weighed the Earth and thus estimated a value for Newton's Universal Gravitational Constant, G. In the process, the mountain became the first place ever mapped with contours. As a student, I myself contributed to the mapping of its Northern flank. The whole district is unspeakably beautiful, especially in the autumn. Schiehallion is the only North European hill that the Ancient Greeks recorded by name.

Figure Eight
Schiehallion from Loch Rannoch
Coordinates: 56°40’1”N:4°5’57”W
Elevation: 1083 meters
Date: pm 23 July 2006

c Goat Fell

Figure Nine
Goat Fell From Brodick
Coordinates: 55°37’34”N:5°11’31”W
Elevation: 874 meters
Date: pm 5 April 1993

d Iona

Dun I is the abrupt rocky crag at 0.90:0.75 whilst the restored Abbey of St Columba is just discernable at 0.58:0.70.

At 0.75:0.25 you can see the head of a curious Atlantic Gray Seal who emerged to inspect our boat. My sister-in-law Oct-Ja threw him some dried anchovies. The photograph is taken at the juncture when a little girl said "I hope he is not run over by the ferry" as the MV Loch Buidhe approached Fionnphort on its shuttle return from Iona. The seal was of course too good a seafarer to be run down, and accompanied us a little part of the way to Staffa.

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ISBN 1-901522-15-6
£9.95 paperback

APPENDIX ONE

The Locational Statistics of Twenty British and Irish Summits

Serial	Summit	Code	Elevation (meters)	Grid Reference	NGR		Latitude			Longitude			Latitude	Longitude	Notes
					X	Y	D	M	S	D	M	S	(decimal)	(decimal)	
1	Harboro Rocks	HAR	379	SK242553	424300	355300	53	5	40	1	38	20	53.09444444	-1.638888889	
2	Schiehallion	SCH	1083	NN714547	271500	754800	56	40	1	4	5	57	56.66694444	-4.099166667	
3	Iona	ION	100	NM284252	128400	725200	56	20	29	6	23	48	56.34138889	-6.396666667	Dun I
4	Arran	GFL	874	NR991415	199100	641600	55	37	34	5	11	31	55.62611111	-5.191944444	Goat Fell
5	Bass Rock	BRK	106.68	NT602874	360300	687400	56	4	41	2	38	23	56.07805556	-2.639722222	
6	Bass Rock (light)	BRL	46	NT601872	360200	687200	56	4	35	2	38	28	56.07638889	-2.641111111	
7	Cairnpapple Hill	CPH	310	NS987717	298700	671700	55	55	40	3	37	22	55.92777778	-3.622777778	
8	Slieve Snaught	SLS	615		-815600	7364400	55	11	37	7	19	38	55.19361111	-7.327222222	Malin
9	Rathlin Island	RAT	134		-691200	7385500	55	18	6	6	12	36	55.30166667	-6.21	
10	Worcestershire Beacon	WOB	425	SO768452	376900	245200	52	6	17	2	20	21	52.10472222	-2.339166667	
11	The Wrekin	WRE	407	SJ628081	362800	308100	52	40	10	2	33	4	52.66944444	-2.551111111	
12	Cader Idris	CAD	893	SH711130	271100	313000	52	41	58	3	54	32	52.69944444	-3.908888889	
13	Snaefell	SNA	621	SC397880	239800	488100	54	15	48	4	27	41	54.26333333	-4.461388889	
14	Snowdon	SNO	1085	SH609543	261000	354400	53	4	7	4	4	34	53.06861111	-4.076111111	
15	Merrick	MER	843	NX427855	242800	585500	55	8	20	4	28	6	55.13888889	-4.468333333	
16	Lugnaquillia Mountain	LUG	926		-719700	6943200	52	58	9	6	27	58	52.96916667	-6.466111111	Wicklow
17	Cannock Chase	CCH	199	SJ994180	399400	318000	52	45	36	2	0	59	52.76	-2.016388889	
18	Leith Hill	LEH	294	TQ139431	513900	143200	51	10	36	0	22	19	51.17666667	-0.371944444	Surrey
19	Ivinghoe Beacon	IVB	230	SP959168	496000	216800	51	50	31	0	36	30	51.84194444	-0.608333333	
20	Cross Fell	CRF	893	NY687343	368800	534300	54	42	11	2	29	10	54.70305556	-2.486111111	

Note: All Latitudes are**North**

Note: All Longitudes are**West**

C=CADASTER

APPENDIX TWO

The Glance Heights of The Twenty British and Irish Summits

	HAR 1	SCH 2	ION 3	GFL 4	BRK 5	BRL 6	CPH 7	SLS 8	RAT 9	WOB 10	WRE 11	CAD 12	SNA 13	SNO 14	MER 15	LUG 16	CCH 17	LEH 18	IVB 19	CRF 20
1 HAR	379	-2859	-4147	-1973	-2000	-2029	-1924	-3262	-2659	120	275	110	-513	152	-1078	-1404	198	-703	-175	-76
2 SCH	-2859	1083	31	618	107	45	297	-481	-253	-4554	-3332	-2834	-569	-2057	380	-2763	-3424	-7790	-6002	-150
3 ION	-4147	31	100	93	-972	-1002	-420	-72	-148	-5453	-4225	-3236	-994	-2461	-219	-2262	-4523	-9334	-7530	-1364
4 GFL	-1973	618	93	874	-124	-165	287	332	76	-3047	-2065	-1331	246	-712	759	-947	-2302	-6217	-4705	97
5 BRK	-2000	107	-972	-124	107	#####	97	-1535	-1008	-3573	-2564	-2415	-713	-1795	-70	-3034	-2550	-6064	-4530	-45
6 BRL	-2029	45	-1002	-165	#####	46	44	-1566	-1037	-3601	-2593	-2444	-745	-1825	-111	-3063	-2578	-6091	-4558	-88
7 CPH	-1924	297	-420	287	97	44	310	-738	-394	-3316	-2312	-1942	-271	-1321	284	-2176	-2392	-6074	-4543	90
8 SLS	-3262	-481	-72	332	-1535	-1566	-738	615	130	-3911	-2951	-1738	-256	-1132	76	-497	-3395	-7659	-6165	-1181
9 RAT	-2659	-253	-148	76	-1008	-1037	-394	130	134	-3472	-2533	-1590	-160	-1014	115	-825	-2870	-6865	-5399	-710
10 WOB	120	-4554	-5453	-3047	-3573	-3601	-3316	-3911	-3472	425	334	306	-1001	205	-1990	-1043	170	-213	27	-989
11 WRE	275	-3332	-4225	-2065	-2564	-2593	-2312	-2951	-2533	334	407	396	-419	384	-1170	-725	177	-629	-195	-368
12 CAD	110	-2834	-3236	-1331	-2415	-2444	-1942	-1738	-1590	306	396	893	130	889	-602	313	132	-1135	-630	-253
13 SNA	-513	-569	-994	246	-713	-745	-271	-256	-160	-1001	-419	130	621	457	529	16	-661	-3341	-2300	378
14 SNO	152	-2057	-2461	-712	-1795	-1825	-1321	-1132	-1014	205	384	889	457	1085	-92	499	121	-1451	-822	125
15 MER	-1078	380	-219	759	-70	-111	284	76	115	-1990	-1170	-602	529	-92	843	-591	-1370	-4703	-3383	507
16 LUG	-1404	-2763	-2262	-947	-3034	-3063	-2176	-497	-825	-1043	-725	313	16	499	-591	926	-1217	-3576	-2835	-1157
17 CCH	198	-3424	-4523	-2302	-2550	-2578	-2392	-3395	-2870	170	177	132	-661	121	-1370	-1217	199	-611	-170	-421
18 LEH	-703	-7790	-9334	-6217	-6064	-6091	-6074	-7659	-6865	-213	-629	-1135	-3341	-1451	-4703	-3576	-611	294	147	-2823
19 IVB	-175	-6002	-7530	-4705	-4530	-4558	-4543	-6165	-5399	27	-195	-630	-2300	-822	-3383	-2835	-170	147	230	-1742
20 CRF	-76	-150	-1364	97	-45	-88	90	-1181	-710	-989	-368	-253	378	125	507	-1157	-421	-2823	-1742	893

Meters

HP=GLANCE HEIGHT

APPENDIX THREE

Derivative Variables for Twenty British and Irish Eminences

Serial	Summit	Code	Elevation (meters)	Grid Reference	Local Radius	Square of Local Radius	Distance to Sea-Level Horizon	
							Line-of-Sight	Ground Arc
1	Harboro Rocks	HAR	379	SK242553	6371328	40593820483584	69493.33	69490.57
2	Schiehallion	SCH	1083	NN714547	6372032	40602791809024	117476.16	117462.85
3	Iona	ION	100	NM284252	6371049	40590265360401	35695.94	35695.56
4	Arran	GFL	874	NR991415	6371823	40600128343329	105532.85	105523.20
5	Bass Rock	BRK	106.68	NT602874	6371056	40590350477660	36868.92	36868.51
6	Bass Rock (light)	BRL	46	NT601872	6370995	40589577290025	24210.11	24209.99
7	Cairnpapple Hill	CPH	310	NS987717	6371259	40592941245081	62849.70	62847.66
8	Slieve Snaught	SLS	615		6371564	40596827806096	88524.83	88519.13
9	Rathlin Island	RAT	134		6371083	40590698592889	41321.09	41320.51
10	Worcestershire Beacon	WOB	425	SO768452	6371374	40594406647876	73589.99	73586.72
11	The Wrekin	WRE	407	SJ628081	6371356	40594177278736	72014.71	72011.64
12	Cader Idris	CAD	893	SH711130	6371842	40600370472964	106673.86	106663.89
13	Snaefell	SNA	621	SC397880	6371570	40596904264900	88955.63	88949.85
14	Snowdon	SNO	1085	SH609543	6372034	40602817297156	117584.59	117571.24
15	Merrick	MER	843	NX427855	6371792	40599733291264	103644.25	103635.11
16	Lugnaquillia Mountain	LUG	926		6371875	40600791015625	108627.14	108616.61
17	Cannock Chase	CCH	199	SJ994180	6371148	40591526837904	50355.51	50354.46
18	Leith Hill	LEH	294	TQ139431	6371243	40592737365049	61206.25	61204.36
19	Ivinghoe Beacon	IVB	230	SP959168	6371179	40591921850041	54135.84	54134.54
20	Cross Fell	CRF	893	NY687343	6371842	40600370472964	106673.86	106663.89

Meters

LOCR=LOCAL RADII

APPENDIX FOUR

Ranges to the Horizon Computed and from Durleston Data

Serial	Given Elevation		Given Elevation (inches)	Given Distance (miles)	Given Elevation (meters)	Given Distance (meters)	T	s	Specific Defect for T	Specific Defect for s
	Feet	Inches								
1	0	8	8	1	0.203200	1609.344000	1609.084744	1609.084711	-0.016109	-0.016111
2	16	7	199	5	5.054600	8046.720000	8025.286486	8025.282241	-0.266363	-0.266416
3	66	4	796	10	20.218400	16093.440000	16050.582522	16050.548564	-0.266304	-0.266515