# The Distributional Characteristics of Lead Mine Yields 

by
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The Logarithm of the Total Production of Lead Concentrate for each mine of Southern Wales was established ${ }^{1}$ and plotted against Rank. A similar exercise was undertaken for the Logarithm of Total Production of Lead Metal in respect of the British prefectures of Shropshire and Montgomeryshire ${ }^{2}$.

Graphs show that the mines group into Statistical Provinces or CLASSES whose plotted points describe polynomial curves definitive of each segregation.

The graph for Southern Wales shows two clear cubic curves for two productive classes plus a third degenerate point plotting the output of a tiny trial. The Shropshire graph also develops three classes: The most prolific follows a cubic curve, the middling class a quadratic and the small mines a linear relation. The Montgomeryshire graph is much more complex. Six or Nine separate classes can be resolved by inspection and the curves displayed are usually cubic or quadratic. It is only in the Montgomeryshire case that the full assemblage approximates the Zipfian paradigm of a linear relationship between the logarithm of size and the distributional rank.

## The Co-ordination of Data

In order to facilitate analysis and comparison it is desirable to perform an elementary transformation of the data to express it in relative terms. This has the twin advantages of making Log Production, T, and Rank, R, both non-dimensional and also normalises each variable to the interval zero to unity.

To maintain a linkage of this distributional data to the original we may record $\mathrm{T}_{\text {max }}$, the Log Production of the Most Prolific Mine, and m , the Number of Mines in the region.

The following equations define the required transformations:-

$$
\begin{equation*}
p_{i}=\frac{T_{\max }-T_{i}}{T_{\max }} \tag{Eqn. 1}
\end{equation*}
$$

so long as $\mathrm{T}_{\mathrm{i}} \geq 0$ and:-

$$
r_{i}=\frac{m-R_{i}}{m-1}
$$

Eqn. 2
It can be seen that under this scheme the most productive mine has ( $\mathrm{p}_{\mathrm{i}}=0: \mathrm{r}_{\mathrm{i}}=1$ ) and the least ( $\mathrm{p}_{\mathrm{i}} \rightarrow 1: \mathrm{r}_{\mathrm{i}}=0$ ).

## A Descriptive Model

Experiments and trials were made with various model components applied to raw data (T,R). Both linear and polynomial regressions were essayed to eliminate trend from regional distributions. The former only eliminated $51 \%$ of variation but the latter some $97 \%$. In both cases residuals tended to describe a damped oscillation which several heuristics based upon the approximate analytic function failed satisfactorily to approximate. It then occurred to me that the worksheet plot of the oscillating residual was disturbingly like Fig 10.1b of "Computational Mathematics" ${ }^{3}$, an expression of the error remaining after fitting a polynomial to data.

It occurred to me that I was chasing chimeras and that I required to return to first principles and contemplate my discovery of the polynomial provinces rather than ignore it.

Taking this view I might summarise the functional predictor of production as:-

$$
T_{i}=X_{\alpha, 0} \cdot P_{i}
$$

Eqn. 3
where Switching Function, X , has these characteristics:-

$$
X(r)=\left\{\begin{array}{c}
1 \text { if } \alpha \leq r \leq \omega  \tag{Eqn. 4}\\
0 \quad \text { elsewhere on the interval }
\end{array}\right.
$$

Modeling the Switching Function
A FOURIER SERIES approximates an arbitrary periodic function subject to the DIRICHLET CONDITIONS ${ }^{4}$ :-

1. Function discontinuities (if any ) are finite in any period
2. The function must contain a finite number of extrema in any period
3. The function must be absolutely integrable in any period

Of especial interest to us is the guarantee and limitation:-
'Suppose that $f(\mathbf{x})$ is a periodic function of period $2 \pi$ and is defined and bounded for $0 \leq x<2 \pi$, and suppose that the interval $(0,2 \pi)$ can be split into finitely many subintervals in each of which the function is continuous and monotonic. Then the Fourier series of $f(x)$ converges at each point of continuity $x_{0}$ to $f\left(x_{0}\right)$, and at a point a jump discontinuity x to the mean value .... of its left and right limiting values". ${ }^{5}$

For Period $=2 \pi / \omega$ the FOURIER SERIES is defined by the equations ${ }^{4}$ :-

$$
\begin{equation*}
P(t)=\frac{1}{2} a_{0}+\sum_{n=1}^{\infty}\left(a_{n} \operatorname{Cosn} \omega \mathrm{t}+\mathrm{b}_{\mathrm{n}} \operatorname{Sin} n \omega \mathrm{t}\right) \tag{Eqn. 5}
\end{equation*}
$$

whilst for $\mathrm{n}=1,2,3, \ldots . .:-$

$$
\begin{array}{ll}
a_{n}=\frac{\omega}{\pi} \int_{-\pi / \omega}^{\tau / \omega} P(t) \operatorname{Cosn} \omega \mathrm{t} . \mathrm{dt} & \text { Eqn. } 6 \\
b_{n}=\frac{\omega}{\pi} \int_{-\pi / \omega}^{\tau / \omega} P(t) \operatorname{Sinn} \omega \mathrm{t} . \mathrm{dt} & \text { Eqn. } 7 \tag{Eqn. 7}
\end{array}
$$

In practical computation the Equation Five summation must be taken to a finite number of series components ( v say ) depending upon the precision of the harmonic synthesis required. The coefficient integrals may be analytic but may in principle be numerical if necessary or convenient.

It is of course the case that we need a shiftable square-wave pulse of sufficient stability within the interval $(0,2 \pi)$.

This pulse is controlled by the Width (w) and $\operatorname{Offset}(\Omega)$ Parameters illustrated below ${ }^{6}$ :-


Figure 1

The relevant Fourier Coefficients are:-

$$
a_{n}=\frac{4 p \ln \frac{1}{2} a_{0}=2 p a}{\operatorname{Sinn} \pi \mathrm{a} \operatorname{Cosn} \pi \mathrm{~b}} \begin{array}{rc}
\mathrm{n} \pi & \text { Eqn. } \mathbf{8} \\
b_{n}=0 & \text { Eqn. } 9 \\
& \text { Eqn. } \mathbf{1 0}
\end{array}
$$

To express the system of Equations Five, Eight, Nine and Ten in computational terms we require to remember that mine output rank is assimilated to the interval $(0,1)$ by Equation Two and that the Width and Offset Metrics a and $b$ need to be expressed co-ordinately.

Accordingly, for $1 \leq i \leq m$ and $1 \leq j \leq v:-$

$$
\begin{gathered}
\frac{1}{2} a_{0}=\frac{R_{\omega}-R_{\alpha}+1}{m} \\
a_{i}=\frac{4 \operatorname{Sin}\left(j \pi \cdot \frac{R_{\omega}-R \alpha+1}{2 m}\right) \operatorname{Cos}\left(j \pi \cdot\left[\frac{R_{\omega}-R \alpha+1}{2 m}+\frac{R \alpha-1}{m}\right]\right)}{j \pi} \\
b_{i}=0
\end{gathered}
$$

Eqn 11

Eqn 12
Eqn 13
If for efficiency and convenience we define the Relative Range

Constant, z, using:-

$$
\begin{equation*}
z=\frac{R_{\omega}-R_{\alpha}+1}{2 m} \tag{Eqn. 14}
\end{equation*}
$$

we may declare the Shifted Square-Wave Fourier Series as:-

$$
X=2 z+\sum_{i=1}^{v} \frac{4 \operatorname{Sinj} \pi z \cdot \operatorname{Cosj} \pi\left(\mathrm{z}+\frac{\mathrm{R}_{\alpha}-1}{\mathrm{~m}}\right) \cdot \operatorname{Cos} \mathrm{j} \pi\left[1-\mathrm{r}_{\mathrm{i}}\right]}{\mathrm{j} \pi}
$$

Eqn. 15

Numerical trials of Equation Fifteen using Program FOURSERI.BAS for a variety of m and v established that for acceptable accuracy v should compare with or exceed m . Special conditions involving few mines in a region ( low $m$ ) could give surprisingly accurate $X$ but in general the penalty of low v was considerable rounding of the range-pulse flanks and averaging ( $\mathrm{p} \approx 0.5$ ) would often persist for $\mathrm{r}_{\omega}$ at the rank of the classe's least productive mine.

Modelling the Polynomial Function
The highest degree delineation so far detected is a Cubic, represented by the general form:-

$$
P=a_{0} r^{0}+a_{l} r^{1}+a_{2} r^{2}+a_{3} r^{3}
$$

Eqn. 16
or:-

$$
\begin{equation*}
P=\prod_{i=0}^{3} a_{i} r^{i} \tag{Eqn. 17}
\end{equation*}
$$

The Cubic Polynomial is general enough to define the Quadratic and Linear Degrees by zeroisation of $\mathrm{a}_{3}$ and $\mathrm{a}_{3}, \mathrm{a}_{2}$ respectively.

## Measures of Fitment Quality

To assess the extent to which the segmentalised polynomial fitments agreed with the original data a series of Ranked Log Productions were synthesized from the Class Polynomial Regression Coefficients, each set of coefficients applied only to the Class Ranks concerned. ( This synthetic series is headed "Logically Switched Model" in the Model Reports *.MOD ).

This series was then compared with the Original Data computed directly from mine histories.

The formula employed was:-

$$
\begin{equation*}
C_{d}=\frac{\sum\left(T_{s m}-\mu_{T}\right)^{2}}{\sum\left(T_{i}-\mu_{T}\right)^{2}} \tag{Eqn. 18}
\end{equation*}
$$

When it came to comparing Fourier Switched Models against this Logically Switched Series the Determination Coefficient Formula of Equation Eighteen proved unsatisfactory.

Clarity required the use of some formula which would measure the residual error in the approximation.

The Fourier Switched Models also embody polynomial syntheses for each Class by utilising the Switching Function of Equation Fifteen to bound the treated Classes.

The chosen Residual Error Formula was:-

$$
\begin{equation*}
\varepsilon=\frac{\sum\left(T_{i}-T_{v}\right)^{2}}{\Sigma T_{i}^{2}} \tag{Eqn. 19}
\end{equation*}
$$

The Residual Error was also computed for the Logically-Switched Series against the Original Data in order to provide a reference metric for adjudging convergence.

## The Organisation of Data

As aforementioned the Lead Mine Populations of three British provinces were analysed:-
(A) SOUTHERN WALES

An area of some $5720 \mathrm{~km}^{2}$ comprising the old Welsh counties of Pembrokeshire, Carmarthenshire, Radnorshire, Breconshire, and the southern fringes of Cardiganshire. In 1974 the Welsh counties were abolished and the larger part of this area is now in Dyfed.

Local hills provide sheep turf, but the preponderant landscape is a low, well-watered and very verdant cow pasture dotted with large villages and small towns.

The typical country rock is Lower Palaeozoic argillite.
(B) SHROPSHIRE

This English county of $3487.581 \mathrm{~km}^{2}$ contained almost all its lead mines on the Shelve Plateau, a lonely tract of scrub and pasture near the Montgomeryshire border.

The country rock is Paleozoic with a Precambrian quartzite
inlier ( The Stiperstones ). None of the former facies are Dinantian blue limestone.

## (C) MONTGOMERYSHIRE

This was a large ( $2064.342 \mathrm{~km}^{2}$ ) county of Central Wales which, in 1974, became an important component of the new Powys prefecture.

Paleozoic argillites predominate in a landscape of grassy hills and silent, sequestered valleys.

I have organised the input and output data into sets of five reports for each province. Each set forms the respective Appendices One, Two and Three.

## Province Report Components

## (1) STATISTICAL PROVINCE GRAPHS

Headed "The Natural Logarithm of Total .... Lead Production ...." these illustrate the segregation of mines into Classes when ranked by Log Production.

Note the two clear cubic provinces on the South Wales plot, the apparent cubic, quadratic and linear orders of the Shropshire rankings, and the several separate cubic or quadratic classes for Montgomeryshire.

The Correlation Coefficients for Montgomeryshire are all based on cubic fitments and in the Shropshire case the four smallest, erratic mines have been suppressed.

The reason why the CC figures on the following text-body plots disagree slightly with those in the text is that I used the EXCEL regression engine to compute the figures in these late internet diagrams, that appear below for reader convenience:-

THE NATURAL LOGARITHM OF
SOUTH WALES LEAD MINE TOTAL METAL PRODUCTION
During the Years 1845 to 1938


THE NATURAL LOGARITHM OF
SHROPSHIRE LEAD MINE TOTAL METAL PRODUCTION
During the Years 1845 to 1913


(2) NORMALISED SOURCE DATA

These worksheet printouts *.CSV list Mine Name, Normalised Rank, and Normalised Log Production for each province.
(3) POLYNOMIAL REGRESSION COEFFICIENTS

The Class Regression Coefficient Files *.COF are shown consolidated upon display reports for each province.

## (4) TABULATED MODEL FITMENTS

These key reports *.MOD compare the quality of both the least-squares polynomial regressions concordance with the source data; and also Fourier-switched modellings with the limiting polynomials' approximation.

The Fourier models were tabulated for $\mathrm{v}=64,128,256,512$ and 1024 .

Summary metrics of correlation are also listed.

## (5) MODEL AGREEMENT GRAPHS

These graphs use the Fitment Series to illustrate the degree of fidelity with which each fitted model follows the curve of the normalised Original Data.

They make three striking demonstrations of the linear convergence of the logically switched and Fourier Series models set against the original data.

For Fourier modeling note how approximations to the data improve as the number of Fourier Series components rises from 64 to 1024.

The Montgomeryshire convergence is perhaps the most impressive of the three.

These Model Agreement Graphs also appear in the Appendices and EXCEL renditions are given below for reader convenience:-

THE SOUTH WALES LEAD MINES
Ranked by The Logarithm of Total Production

-Original Data - Logically Switched -Fourier 64 -Fourier 128 -Fourier 256 -Fourier 512 -Fourier 1024
THE SHROPSHIRE LEAD MINES
Ranked by The Logarithm of Total Production


[^0]
-Original Data - Logically Switched -Fourier 64 -Fourier 128 -Fourier 256 -Fourier 512 -Fourier 1024
Program POLYWORK.BAS
Appendix Four records the source code of POLYWORK.BAS whose first option was invoked to generate for each Class the Least Squares Polynomial Regression coefficient files *.COF.

## Program MINEMODL.BAS

Appendix Five contains the MINEMODL.BAS source code. This program utilised the coefficient files *.COF with Class and Fourier definitional keyboard data to generate the model fitments *.MOD.

## A Review of Results

Southern Wales
The Statistical Province Graph illustrates the superb segregation of the lead mines into cubic Classes or Statistical Provinces. I admit that the West Nantymwyn trial could conform to any function but there is little reason why we should not allow it to qualify as a degenerate cubic!

Sensationally, the Second Class Cubic Fitment accounts for
$99.99993 \%$ of the data variation whilst the First Class Fitment accounted for $99.87 \%$. Readers should place scant credence upon the Third Class fit quality as it is entirely notional.

The Determination due to polynomial approximations is $99.94 \%$.
The best Fourier fitments are $\mathrm{v}=64$ and $\mathrm{v}=512$ whilst the Model Agreement Graph shows good approximations for middle ranks though low-v fits wander at high and low extremes.

Shropshire
The Statistical Province graph shows another interesting three-class segregation, this time into a Cubic First, a Quadratic Second and a Linear Third Class. It is of course true that Quadratic and Linear polynomials are special cases of the Cubic.

Fitments are not significantly inferior to $\mathrm{C}_{\mathrm{d}}=99 \%$ for any class and the Determination due to polynomial approximations is $99.86 \%$.

Fourier fitments improve with steady convergence until $\mathrm{v}=1024$ ( and perhaps beyond? ) when the residual error is measurable in millionths.

The Model Agreement Graph shows Fourier fitments are generally excellent for the First and Second Classes though low-v series diverge from the required line for the Third Class.

Montgomeryshire
The mine population of Montgomeryshire is large but is characterised by three or four large mines dominating a province in which fifty very middling workings show very similar low yields.

In any event, the simple elegance of the Southern Wales and Shropshire distributions does not apply and the Montgomeryshire field resolves into either six or nine indistinct Classes. The chosen study illustrates six Cubic Classes.

For Classes Two and Four $\mathrm{C}_{\mathrm{d}}$ 's are as low as $94 \%$ but the overall Determination due to polynomial approximations is, however, $99.9 \%$.

Fourier fitments are of poor quality and even that for $\mathrm{v}=1024$ shows a residual error an order of magnitude away from the logically-switched series.

Study of the Model Agreement Graph discloses Fourier fitments radically more ill-behaved than those for either Southern Wales or Shropshire and characterised by unstable oscillations across the field, especially at the extremes.

I have neither a compiler nor a math co-processor on my old 20 MHz

Tiko 386. Accordingly MINEMODL.BAS was elaborated as interpreted source. Both the Southern Wales and the Shropshire modeling took about half an hour to complete and I guessed the Montgomery exercise should take an hour and a quarter on that basis: It took nearly two hours to produce an inferior result, though for unknown reasons.

## The Absolute Magnitude of Total Mine Productions

This study is of course couched in relative terms in order to facilitate inter-provincial comparisons and dimensionless computations.

If you wish to re-construct absolute data you will need figures for the Total Production of Metal or Concentrate for a defined mine in each province.

In practical terms, Table One specifies accumulated Production in Metric Tonnes for the leading mine:-

| Province | Mine Name | Production | Substance |
| :--- | :--- | :--- | :--- |
| Southern Wales | Nantymwyn | 34888.0571386 | Concentrate |
| Shropshire | Snailbeach | 91302.51954 | Metal |
| Montgomeryshire | Van | 67479.84826 | Metal |

A Formal Statement of The Cubic Class Rule

Equation One implies that:-

$$
p_{i}=\frac{\log _{\mathrm{n}}\left(\mathrm{Y}_{\max }\right)-\log _{\mathrm{n}}\left(\mathrm{Y}_{\mathrm{i}}\right)}{\log _{\mathrm{n}}\left(\mathrm{Y}_{\max }\right)}=1-\frac{\log _{\mathrm{n}}\left(\mathrm{Y}_{\mathrm{i}}\right)}{\log _{\mathrm{n}}\left(\mathrm{Y}_{\max }\right)}
$$

Eqn. 20

I have demonstrated from recorded data that:-

$$
\begin{equation*}
p_{i, k}=a_{0, k}+a_{l, k} r_{i}+a_{2, k} r_{i}^{2}+a_{3, k} r_{i}^{3} \tag{Eqn. 21}
\end{equation*}
$$

For Class k let:-

$$
\begin{equation*}
a_{k}^{\prime}=1-a_{0, k} \tag{Eqn. 22}
\end{equation*}
$$

Then it follows that:-

$$
\begin{gather*}
\frac{\log _{n}\left(Y_{i}\right)}{\log _{n}\left(Y_{\max }\right)}=1-a_{0, k}-a_{1, k} r_{i}-a_{2, k} r_{i}^{2}-a_{3, k} r_{i}^{3}  \tag{Eqn. 23}\\
=a_{k}^{\prime}-\left(a_{1, k} r+a_{2, k} r^{2}+a_{3, k} r^{3}\right)
\end{gather*}
$$

or:-

$$
\begin{equation*}
\frac{\log _{n}\left(Y_{i}\right)}{\log _{n}\left(Y_{\max }\right)}=a_{k}^{\prime}-U_{k} \tag{Eqn. 24}
\end{equation*}
$$

## Discussion

The Number of Fourier Series Components, v
When I ran the Model Fitments I incremented v in steps of $2{ }^{i}$. As a doctoral student I learned that several relatively-efficient computational schemes ( such as some FFT's ) depended upon contrived or natural data populations of extent $2{ }^{\text {i }}$.

The computational scheme of MINEMODL.BAS does not require this restriction but I nevertheless considered it good policy to use $2^{i}$ increments in case it should facilitate extended operations.

The Mixed Character of Modelling
Equation Three defines the structure of the mixed Fourier Series Polynomial Regressions models I have essayed.

By now many readers will be wondering why I did not discuss the logically-switched model alone. To be sure, the determinate action of the bistable multivibrator sets the limiting accuracy of the logically-switched model to the collective fidelity of the least-squares regressions. And the program has shown that Determination to exceed $99.8 \%$ in each province. I guess that many practical applications of The Cubic Class Rule could be made using only logicallyswitched modelling.

My motive for using the hybrid is that several orthogonal series of generated terms populate a multidimensional matrix which you may sum across many different vectors in order to create different expressions of Equation Three which may facilitate both theoretical and computational analysis of the underlying statistical situation.

Many other readers will take the contrary tack and wonder why I did not discountenance the regressions and develop pure Fourier Series models for the entirety of the ranked productions in each province.

Aside from my very uncertain skills at the calculus proving the usual handicap a more respectable reason for rejecting the pure model is that it would ignore The Cubic Class Rule and once again I needed to contemplate that discovery rather than ignore it.

## Residual Errors in Modelling

Cursory examination of the five-member Residual Error series for each province suggests that such error may conform to a damped oscillatory rule of the kind typical for series errors.

The Selection of $v$

As aforementioned, Tabulated Model Fitments were computed for v $=64,128,256,512$ and 1024 or in general:-

$$
v=2^{i}
$$

Eqn. 25
where $\mathrm{i}=6,7,8,9$ and 10 .
Note that for even $\mathrm{i} v$ is a perfect square defined by:-

$$
v=2^{2 j}
$$

Eqn. 26
where $\mathrm{j}=3$, 4 and 5
and the Square Root of $v$ is given by:-

$$
\begin{equation*}
\sqrt{v}=2^{j} \tag{Eqn. 27}
\end{equation*}
$$

This is of interest because there is a numerical rule, valid in many instances, which asserts that the most accurate sum of a series is determined when it can be and is summed as $n$ ranks of $n$ separately-totalled data elements. This rule is of course predicated upon having a data strip whose population is a perfect square, a condition satisfied for alternate series in this study.

Such a summation may formally be defined by:-

$$
S=\sum_{i=1}^{2^{i}} \sum_{j=1}^{2^{j}} x_{i, j}
$$

Eqn. 28

Reference to the Tabulated Model Fitments confirms that Model Error Metrics for even i are not generally inferior to those of odd i. I therefore consider that savings may be made by computing even i v's only and accuracy enhanced by re-configuring the computational program to execute a two-phase summation according to Equation Twenty-Eight.

## Further Research

Though I am a trained geologist I have no idea why this Cubic Class Rule should apply to the yield distributions of Victorian lead mines in and about the British Lower Palaeozoic.

An obvious diversification would explore the relation both for other kinds of mine and for the more typical British lead mines in the Dinantian blue limestone.

It is very clear that The Zipfian Paradigm of the linear relation of Size to Rank is violated by these objects.

I suspect that the observed distributions are as much the result of socio-environmental as of scientific factors and the published Victorian and Edwardian records already furnish some statistics, for instance regarding employment, prices and continuity of operation, which may supply multiple regression and other correlative studies with the data to resolve the relevant determinants of productivity.

Nevertheless, some of the data collated by The University of Exeter and published in computer-composited handbooks should be treated with circumspection. I used the occasional grid references of the Shropshire mines to map them by Class. It was clear from the fall of the plotted position that many coordinates were notional or highly-inaccurate. Allowing for that there was, however, clearly no correlation of Class with location within that province.

In the Dinantian region of Derbyshire a great many non-yielding mines were registered in the nineteenth-century for apparently fiscal motives ${ }^{7}$ and except for two or three high-yielders only desultory lead mining took place during the period.

## Notation

$\alpha \quad$ The Lower Bounding Rank of Class k
a A Pulse Half-Width
$\mathrm{a}_{0} \quad$ The Fourier Series Constant Coefficient
$a_{i} \quad$ A Polynomial Equation Coefficient
$\mathrm{a}_{\mathrm{i}, \mathrm{k}} \quad$ The Polynomial Equation Coefficient for the ith. Term of Class k
$a_{n} \quad$ The First Fourier Series Coefficient
b A Pulse Shift
$b_{n} \quad$ The Second Fourier Series Coefficient
$\mathrm{C}_{\mathrm{d}} \quad$ The Coefficient of Determination
$\varepsilon \quad$ The Residual Error in Approximating Two Series
i The First Series Counter
j The Second Series Counter
k The Number of Mine Classes in the Region
L A Fourier Series Period
$\mu_{\mathrm{T}} \quad$ The Mean of The Logarithms of Total Production
$\mathrm{m} \quad$ The Number of Mines in the Region
v The Number of Fourier Series Components
n The Number of Data
n The Number of Polynomial Equation Terms
$\pi \quad$ The Ludolphine Constant
p A Pulse Height
$\mathrm{p}_{\mathrm{i}} \quad$ The Normalised Logarithm of Total Production for Mine i
P A Generalised Polynomial Function
$\mathrm{P}(\mathrm{t}) \quad$ A Generalised Fourier Series Function
$\mathrm{r}_{\mathrm{i}} \quad$ The Normalised Distributional Rank
$\mathrm{R}_{\alpha} \quad$ The Rank of the Class Most Productive Mine
$\mathrm{R}_{\omega} \quad$ The Rank of the Class Least Productive Mine
$\mathrm{R}_{\mathrm{i}} \quad$ The Distributional Rank
S A Series Sum
T The Napierian Logarithm of Total Production
$\mathrm{T}_{\mathrm{i}} \quad$ The Total Production of the ith. Mine
$\mathrm{T}_{\text {max }} \quad$ The Total Production of the Most Prolific Mine
$\mathrm{T}_{\mathrm{v}} \quad$ A Fourier-Switched Log Production Computed from Polynomial Coefficients
$\mathrm{T}_{\mathrm{Pb}} \quad$ The Napierian Logarithm of the Total Production of Metallic Lead
$\mathrm{T}_{\mathrm{PbS}}$ The Napierian Logarithm of the Total Production of Lead Ore Concentrate
$\mathrm{T}_{\text {syn }}$ A (Logically-Switched) Log Production computed from Polynomial Coefficients
$\mathrm{U}_{\mathrm{k}} \quad$ The Origin-Interceptive Cubic $\mathrm{a}_{1, k} \mathrm{r}^{1}+\mathrm{a}_{2, k} \mathrm{r}^{2}+\mathrm{a}_{3, k} \mathrm{r}^{3}$ for Class k
w The Fourier Series Square-Pulse Width
x An Arbitrary Datum
X A Switching Function
Y The Total Production in Metric Tonnes
$\mathrm{Y}_{\mathrm{i}, \mathrm{k}}$ The Total Production of Mine i , a member of Class $k$
$\mathrm{Y}_{\text {max }}$ The Total Production of the Leading Mine
$\omega \quad$ The Upper Bounding Rank of Class k
$\Omega \quad$ The Fourier Square-Wave Offset

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## APPENDIX ONE

Province Report for Southern Wales



## SWALES.CSV

THE SOUTH WALES LEAD MINES
Ranked by The Logarithm of Total Production
Normalised Rank
Normalised Production
18
NANTYMWYN
LLANFYRNACH
TOWY VALE
CYSTANOG
NANTYGARW
LLANFAIR
CASARA
CARMARTEN
NANTYCAR
PENEGARREG
CWM ELAN
FEDW
DALRHIW
BRYNAMBOR
ST DAVIDS
WHEALMORGAN
ABERGWESSIN
W NANTYMWYN
1.0000000
.9411765 .0851440
.8823529 .1710357
.8235294 . 2480279
.7647059 .2720017
.7058824 . 3253142
.6470588 . 3567280
.5882353 . 3749819
.5294118 . 4199669
.4705882 .4389746
.4117647 . 4720195
.3529412 .4908477
.2941176 .6333156
.2352941 . 7072130
.1764706 . 7275456
.1176471 . 7434905
.0588235 . 8030330
.00000001 .0000000

```
    SWALE1.CSV
FIRST CLASS SOUTH WALES LEAD MINES
Cubic Segment
Normalised Rank
Normalised Production
```

|  | 4 |  |
| :--- | ---: | ---: |
|  | 0 | .8963591 |
|  | 1 | -1.9213202 |
|  | 2 | 2.8915149 |
| COEFFICIENT OF DETERMINATION $=$ | 3 | -1.8677488 |
| COEFFICIENT OF CORRELATION $=$ | .9973554 |  |
| STANDARD ERROR OF THE ESTIMATE $=$ | .9986768 |  |
|  |  | .0093637 |

SWALE2.CSV
SECOND CLASS SOUTH WALES LEAD MINES Cubic Segment

Normalised Rank

Normalised Production

4
0.9551120

1 -3.6460390
2 20.3767520
3-39.7799635
COEFFICIENT OF DETERMINATION $=$
. 9999987
COEFFICIENT OF CORRELATION =
. 9999993
STANDARD ERROR OF THE ESTIMATE =
.0001425

SWALE3.CSV
THIRD CLASS SOUTH WALES LEAD MINE
Cubic Segment with a Single Member
Normalised Rank
Normalised Production
4
01.0000000
1.0000000
2.0000000

3
.0000000
1.0000000

COEFFICIENT OF DETERMINATION =
1.0000000

STANDARD ERROR OF THE ESTIMATE =
.0000000

A WORY MODEL FITMENTS TO LEAD MINE
LOG PRODUCTION VERSUS RANK DATA

JAMES $R$ WARREN
SWALES.MOD
07-11-1995

THE SOUTH WALES LEAD MINES
Ranked by The Logarithm of Total production
NUMBER OF CLASSES 3

| ORIGINAL DATA | SWALES.CSV |  |
| :--- | :--- | :--- |
| NUMBER OF MINES | 18 |  |

POLYNOMIAL SEGMENTS

| REGRESSION | LOWER | UPPER |  |
| :--- | :--- | :---: | :---: |
| FILENAME | RANK | RANK |  |
| SWALE1.COF |  | 1 | 12 |
| SWALE2.COF |  | 13 | 17 |
| SWALE3.COF |  | 18 | 18 |

THE TABLE OF MODEL APPROXIMATIONS

|  | MINE NAME | ORIGINAL DATA | LOGICALLY SWITCHED MODEL | FOURIER <br> SWITCHED |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| NUMBER OF TERMS = |  |  |  | 64 | 128 | 256 | 512 | 1024 |
|  | NANTYMWYN | . 000000 | - -. 001195 | . 014208 | -. 056954 | . 050809 | . 005234 | . 008595 |
|  | LLANFYRNACH | . 085144 | . 092245 | . 101863 | . 081969 | . 051843 | . 097183 | . 097531 |
|  | TOWY VALE | . 171036 | - . 169200 | . 171673 | . 201690 | . 197952 | . 172497 | . 168541 |
|  | CYSTANOG | . 248028 | - 231951 | . 228804 | . 250510 | . 213692 | . 233719 | . 227863 |
|  | NANTYGARW | . 272002 | -. 282778 | . 276969 | . 268953 | . 292438 | . 283295 | .278874 |
|  | LIANFAIR | . 325314 | - . 323963 | . 318472 | . 307292 | . 320692 | . 323579 | . 322409 |
|  | CASARA | . 356728 | - . 357787 | . 354563 | . 360045 | . 356862 | . 356853 | . 358526 |
|  | CARMARTEN | . 374982 | . 386530 | . 386013 | . 397218 | . 389729 | . 385354 | . 388164 |
|  | NANTYCAR | . 419967 | - 412474 | . 413761 | . 415162 | . 408501 | . 411298 | . 413659 |
|  | PENEGARREG | . 438975 | 5 . 437900 | . 439457 | . 433109 | . 441634 | . 436904 | . 438170 |
|  | CWM ELAN | . 472019 | - 465088 | . 465660 | . 462057 | . 462150 | . 464406 | . 464775 |
|  | FEDW | . 490848 | - . 496319 | . 495189 | . 497416 | . 498032 | . 496171 | . 496030 |
|  | DALRAIW | . 633316 | . 633333 | . 632569 | . 635655 | . 631656 | . 633229 | . 633099 |
|  | BRYNAMBOR | . 707213 | 3.707145 | . 708054 | . 706704 | . 708266 | . 707479 | . 707170 |
|  | ST DAVIDS | . 727546 | -.727648 | . 731723 | . 726624 | . 726620 | . 728309 | . 727905 |
|  | WHEALMORGAN | . 743490 | - . 743422 | . 751782 | . 745678 | . 744883 | . 744351 | . 743596 |
|  | ABERGWESSIN | .803033 | . .803050 | . 862124 | . 827193 | . 788593 | . 807406 | $.800022$ |
|  | W NANTYMWYN | 1.000000 | 1.000000 | . 999568 | 1.001427 | . 999539 | 1.000032 | 1.000154 |
| DETERMINATION COEF. |  | 1.000000 | .999419 |  |  |  |  |  |
| MODEL ERROR METRICS |  | . 000000 | . 000140 | . 000944 | . 001166 | . 001349 | .000161 | . 000200 |



## APPENDIX TWO

Province Report for Shropshire


SHROP.CSV
THE SHROPSHIRE LEAD MINES
Ranked by The Logarithm of Total Production
Normalised Rank
Normalised Production
SNAILBEACH
ROMAN GRAVELS
TANKERVILLE
PENNERLEY
STIPERSTONES
WHITE GRIT
BOG
GRIT
ROUND HILL
ROMAN GRAVELS EAST
TANKERVILLE WEST
OVENPIPE
PERKINS BEACH
BATHOLES
LADYWELL
STAPELEY
HOPE VALLEY
RORRINGTON
LEEDS ROCK HOUSE
BOG SOUTH
SALOP SOUTH
POTTERS PIT
CRICKHEATH
STAVELEY
BURGAM
LORD HILL
TANKERVILLE NORTH
ROMAN GRAVELS SOUTH
RHADLEY
WHITE GRIT EAST
BATHOLES OLD
ROMAN GRAVELS WEST
SNAILBEACH NEW WEST
CALLOW HILL

| 1.0000000 | .0000000 |
| ---: | ---: |
| .9696970 | .0812912 |
| .9393939 | .1772952 |
| .9090909 | .2560921 |
| .8787879 | .2790697 |
| .8484848 | .2953760 |
| .8181818 | .3078523 |
| .7878788 | .3273448 |
| .7575758 | .3274422 |
| .7272727 | .3468171 |
| .6969697 | .3686854 |
| .6666667 | .4186914 |
| .6363636 | .4327732 |
| .6060606 | .4371849 |
| .5757576 | .4390224 |
| .5454545 | .4482985 |
| .5151515 | .4930467 |
| .4848485 | .5297870 |
| .4545455 | .5734661 |
| .4242424 | .5862231 |
| .3939394 | .6046756 |
| .3636364 | .6256609 |
| .3333333 | .6478107 |
| .3030303 | .7018938 |
| .2727273 | .7034478 |
| .2424242 | .7357035 |
| .2121212 | .7495690 |
| .1818182 | .8064113 |
| .1515152 | .8092578 |
| .1212121 | .8310746 |
| .0909091 | .8675223 |
| .0606061 | .8796551 |
| .0303030 | .8873666 |
| .0000000 | .9296027 |

```
    SHROP1.CSV
FIRST CLASS SHROPSHIRE LEAD MINES
Cubic Segment
Normalised Rank
Normalised Production
```


## 4

015.8442245

1 -58.2147732
273.1043996

3-30.7449865
COEFFICIENT OF DETERMINATION $=$ . 9943634
COEFFICIENT OF CORRELATION = . 9971777
STANDARD ERROR OF THE ESTIMATE = .0107347

SHROP2.CSV
SECOND CLASS SHROPSHIRE LEAD MINES
Quadratic Segment
Normalised Rank
Normalised Production

3
$0 \quad 2.5872400$
1 -7.0276515
25.7275660

COEFFICIENT OF DETERMINATION $=$. 9898382
COEFFICIENT OF CORRELATION = . 9949061
STANDARD ERROR OF THE ESTIMATE = .0067860

SHROP3.CSV
THIRD CLASS SHROPSHIRE LEAD MINES Linear Segment
Normalised Rank
Normalised Production
2
$0 \quad .9306872$
$1-.8153935$
COEFFICIENT OF DETERMINATION = . 9901369
COEFFICIENT OF CORRELATION =
. 9950562
STANDARD ERROR OF THE ESTIMATE =
.0114451

```
A WORKSHEET TO PRESENT FOURIER-POLYNOMIAL
    MODEL FITMENTS TO LEAD MINE
    LOG PRODUCTION VERSUS RANK DATA
LOG PRODUCTION VERSUS RANK DATA
JAMES R WARREN
HROP.MOD
07-11-1995
THE SHROPSHIRE LEAD MINES
Ranked by The Logarithm of Total Production
NUMBER OF CLASSES 3
ORIGINAL DATA SHROP.CSV
POLYNOMIAL SEGMENTS
\begin{tabular}{llcc} 
REGRESSION & \(\begin{array}{l}\text { LOWER } \\
\text { FILENAME }\end{array}\) & \multicolumn{2}{c}{ UPPER } \\
RANK & & \\
RANK
\end{tabular}\(]\)
THE TABLE OF MODEL APPROXIMATIONS
\begin{tabular}{|c|c|c|c|c|c|c|c|c|}
\hline \multirow[b]{2}{*}{NUMBER OF TERMS \(=\)} & MINE NAME & ORIGINAL DATA & \begin{tabular}{l}
LOGICALLY \\
SWITCHED MODEL
\end{tabular} & \begin{tabular}{l}
FOURIER \\
SWITCHED
\end{tabular} & & & & \\
\hline & & & & 64 & 128 & 256 & 512 & 1024 \\
\hline & SNAILBEACH & . 000000 & - -. 011136 & -. 027475 & -. 016999 & -. 010486 & -. 012864 & -. 010905 \\
\hline & ROMAN GRAVELS & . 081291 & 1.100666 & . 086953 & . 096058 & . 100941 & . 100525 & . 100486 \\
\hline & TANKERVILLE & . 177295 & -. 182466 & . 171043 & . 179421 & . 182125 & . 183641 & . 182589 \\
\hline & PENNERLEY & . 256092 & . 239399 & . 229963 & . 237920 & . 238712 & . 239688 & . 239334 \\
\hline & STIPERSTONES & . 279070 & - 276597 & . 268866 & . 276484 & . 276062 & . 275802 & . 276610 \\
\hline & WHITE GRIT & . 295376 & -. 299193 & . 292902 & . 300126 & . 299177 & . 298847 & . 299228 \\
\hline & BOG & . 307852 & . 312321 & . 307218 & . 313931 & . 312844 & . 312858 & . 312246 \\
\hline & GRIT & . 327345 & - . 321114 & . 316963 & . 323031 & . 321864 & . 321490 & . 321225 \\
\hline & ROUND HILL & . 327442 & . 330704 & . 327287 & . 332586 & . 331235 & . 330356 & . 330563 \\
\hline & ROMAN GRAVELS EAST & . 346817 & . 346226 & . 343351 & . 347775 & . 346212 & . 345825 & . 346394 \\
\hline & TANKERVILLE WEST & . 368685 & . 372811 & . 370339 & . 373801 & . 372247 & . 372995 & . 372625 \\
\hline & OVENPIPE & . 418691 & 1.415594 & . 413717 & . 415954 & . 414912 & . 415926 & . 415763 \\
\hline & PERRINS BEACH & . 432773 & 3 . 434526 & . 432974 & . 432794 & . 433462 & . 434423 & .434099 \\
\hline & BATHOLES & . 437185 & -. 431846 & . 429776 & . 429766 & . 431806 & . 431215 & . 432198 \\
\hline & LADYWELL & . 439022 & -. 439686 & . 437597 & . 436830 & . 440655 & . 439526 & . 439319 \\
\hline & STAPELEY & . 448298 & . 458045 & . 456044 & . 454600 & . 459736 & . 458669 & . 458436 \\
\hline & HOPE VALLEY & . 493047 & 7.486922 & . 485181 & . 483257 & . 488618 & . 487316 & . 486508 \\
\hline & RORRINGTON & . 529787 & 7 . 526319 & . 525216 & . 522900 & . 527110 & . 525695 & . 526757 \\
\hline & LEEDS ROCK HOUSE & . 573466 & -. 576234 & . 577409 & . 573133 & . 575464 & . 575845 & . 575721 \\
\hline & BOG SOUTH & . 586223 & 3.584763 & . 581223 & . 585792 & . 582661 & . 585035 & . 585036 \\
\hline & SALOP SOUTH & . 604676 & -.609472 & . 608462 & . 612175 & . 606662 & . 610714 & . 609184 \\
\hline & POTTERS PIT & . 625661 & . 634181 & . 634907 & . 639596 & . 632280 & . 634051 & . 634403 \\
\hline & CRICRHEATE & . 647811 & 1.658889 & . 661565 & . 667226 & . 659382 & . 657368 & . 658777 \\
\hline & Staveley & . 701894 & 4.683598 & . 688610 & . 694562 & . 687026 & . 683315 & . 683558 \\
\hline & BURGAM & . 703448 & . 708307 & . 716115 & . 721068 & . 713678 & . 710099 & . 708547 \\
\hline & LORD HILL & . 735703 & - . 733016 & . 744118 & . 746207 & . 737906 & . 733912 & . 732529 \\
\hline & TANKERVILLE NORTH & . 749569 & . 757725 & . 772637 & . 769513 & . 759239 & . 755784 & . 758510 \\
\hline & ROMAN GRAVELS SOUTH & . 806411 & 1.782434 & . 801669 & .790679 & . 778728 & . 780724 & . 781303 \\
\hline & RHADLEY & . 809258 & 8.807143 & . 831197 & . 809645 & . 798783 & . 809017 & . 808666 \\
\hline & WHITE GRIT EAST & . 831075 & -.831852 & . 861182 & . 826667 & . 822184 & . 834559 & . 829897 \\
\hline & BATHOLES OLD & . 867522 & 2.856561 & . 891563 & . 842356 & . 850601 & . 855080 & . 858978 \\
\hline & ROMAN GRAVELS WEST & . 879655 & . 881269 & . 922259 & . 857678 & . 883345 & . 877430 & . 878370 \\
\hline & SNAILBEACH NEW WEST & . 887367 & 7 . 905978 & . 953162 & . 873896 & . 917111 & . 906614 & . 909364 \\
\hline & CALIOW HILL & . 929603 & \(3 \quad .930687\) & . 984135 & . 892467 & .947137 & . 935708 & . 926833 \\
\hline DETERMINATION COEF. & & 1.000000 & - . 998588 & & & & & \\
\hline MODEL ERROR METRICS & & . 000000 & . 000246 & . 001256 & . 000497 & . 000368 & . 000266 & . 000260 \\
\hline
\end{tabular}
```

uoṭonpoxd pasṭteuxion

## APPENDIX THREE

Province Report for Montgomeryshire



| MONT.CSV |  |  |
| :---: | :---: | :---: |
| THE MONTGOMERYSHIRE LEAD MINES |  |  |
| Ranked by The Logarithm of Total ProductionNormalised Rank |  |  |
|  |  |  |
| Normalised Production |  |  |
|  | 53 |  |
| VAN | 1.0000000 | . 0000000 |
| DYLIFFE | . 9807692 | . 1026285 |
| LLANERCHYRAUR | . 9615385 | . 2323050 |
| DYFNGWM | . 9423077 | . 2640170 |
| MACHYNLLETH | . 9230769 | . 2697373 |
| VAN CONSOLS | . 9038462 | . 2821909 |
| RHOSWYDDOL | . 8846154 | . 2855862 |
| LLANGYNOG | . 8653846 | . 3169295 |
| NANTY | . 8461538 | . 3603565 |
| WYE VALLEY | . 8269231 | . 3639355 |
| CAE CONROY | . 8076923 | . 3671545 |
| BRYNTAIL | . 7884615 | . 3706166 |
| NANTIAGO | . 7692308 | . 3709772 |
| CHIRK CASTLE | . 7500000 | . 3776816 |
| LLANGYNOG NEW | . 7307692 | . 3840458 |
| LLANGYNOG EAST | . 7115385 | . 4211464 |
| PENYCLYN | . 6923077 | . 4229186 |
| LLANGYNOG UNITED | . 6730769 | . 4329949 |
| VAN GREAT WEST | . 6538462 | . 4634831 |
| CWMBYR | . 6346154 | . 4713034 |
| BLAEN TWYMYN | . 6153846 | . 4770017 |
| BRYNPOSTIG | . 5961538 | . 4855012 |
| WYE VALLEY WEST | . 5769231 | . 4957970 |
| RHIWARTH | . 5576923 | . 4963600 |
| GORN | . 5384615 | . 5065198 |
| LLANIDLOES | . 5192308 | . 5126329 |
| NANTY WEST | . 5000000 | . 5294537 |
| CAELAN | . 4807692 | . 5368477 |
| CEFNMAENLLWYD | . 4615385 | . 5868118 |
| CRAIG RHIWARTH | . 4423077 | . 5885599 |
| ABERDAUNANT | . 4230769 | . 5986421 |
| TYISA | . 4038462 | . 5988399 |
| CYFARTHA | . 3846154 | . 6035171 |
| SNOW BROOK | . 3653846 | . 6105747 |
| LLANRHAIADR | . 3461538 | . 6411381 |
| MID-WALES | . 3269231 | . 6443803 |
| NANTMELYN | . 3076923 | . 6856234 |
| CYMOROG | . 2884615 | . 6882848 |
| PANTMAWR | . 2692308 | . 6998041 |
| SEGLENLAS | . 2500000 | . 7185520 |
| CWMVRON | . 2307692 | . 7320420 |
| VAN EAST | . 2115385 | . 7474178 |
| LLANGYNOG NORTH AND SOUTH | . 1923077 | . 7528225 |
| GLYN | . 1730769 | . 7545099 |
| WYE | . 1538462 | . 7603772 |
| CWMRICKET | . 1346154 | . 7777296 |
| BWLCH CREOLAN | . 1153846 | . 7854198 |
| FRONTBALLAN | . 0961538 | . 8265212 |
| PENRALT | . 0769231 | . 8331280 |
| FRONVELLAN | . 0576923 | . 8659108 |
| BRYNYFEDWEN | . 0384615 | . 8764112 |
| SEVERN WATER | . 0192308 | . 9020582 |
| SEVERN | . 0000000 | . 9837460 |




THE TABLE OF MODEL APPROXIMATIONS

| NUMBER OF TERMS = | MINE <br> NAME | ORIGINAL DATA | LOGICALLY <br> SWITCHED MODEL | FOURIER <br> SWITCHED |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  | 64 | 128 | 256 | 512 | 1024 |
|  | VAN | . 000000 | -. 009605 | 1.121817 | -. 712757 | . 033719 | -. 019791 | -. 004281 |
|  | DYLIFFE | . 102628 | . 129493 | -. 646181 | . 070106 | . 089960 | . 121456 | . 132810 |
|  | LLANERCHYRAUR | . 232305 | . 213972 | . 280489 | . 823385 | . 247448 | . 210020 | . 213258 |
|  | DYFNGWM | . 264017 | . 258035 | . 850882 | . 412898 | . 232177 | . 258846 | . 254109 |
|  | MACHYNLLETH | . 269737 | . 275882 | -. 596257 | . 234508 | . 293244 | . 280887 | . 271622 |
|  | VAN CONSOLS | . 282191 | . 281717 | . 955116 | . 059406 | . 273095 | . 289351 | . 280013 |
|  | RHOSWYDDOL | . 285586 | . 289742 | . 135699 | . 700827 | . 289940 | . 297877 | . 291552 |
|  | LLANGYNOG | . 316929 | . 314159 | -. 068315 | .579482 | . 321626 | . 320630 | . 317929 |
|  | NANTY | . 360356 | . 363280 | 1.019838 | . 047896 | . 350356 | . 366238 | . 366404 |
|  | WYE VALLEY | . 363935 | . 361918 | -. 204987 | . 075633 | . 380310 | . 360640 | . 362016 |
|  | CAE CONROY | . 367155 | . 363388 | . 556586 | . 588775 | . 341791 | . 357989 | . 360888 |
|  | BRYNTAIL | . 370617 | . 367501 | . 602873 | . 657781 | . 390463 | . 359163 | . 364607 |
|  | NANTIAGO | . 370977 | . 374071 | -. 116340 | . 229824 | . 351556 | . 364765 | . 373014 |
|  | Chirk castle | .377682 | . 382907 | . 842544 | . 103540 | . 403258 | . 374979 | . 384136 |
|  | LIANGYNOG NEW | . 384046 | .393824 | . 201362 | . 466589 | .377183 | . 389463 | . 395824 |
|  | LLANGYNOG EAST | . 421146 | . 406631 | . 263326 | . 663218 | . 418253 | . 407353 | . 407497 |
|  | PENYCLYN | . 422919 | . 421142 | . 783388 | . 409483 | . 415564 | . 427369 | . 420405 |
|  | LLANGYNOG UNITED | . 432995 | . 437167 | . 076042 | . 212666 | . 435925 | .448011 | . 436094 |
|  | VAN great west | . 463483 | . .463160 | . 627065 | . 425946 | . 470799 | . 476719 | . 463530 |
|  | CWMBYR | . 471303 | . 471359 | . 567448 | . 657205 | . 456657 | . 484371 | . 472779 |
|  | BLAEN THYMYN | . 477002 | . 478607 | . 205849 | . 555812 | . 499688 | . 487894 | . 479291 |
|  | BRYNPOSTIG | . 485501 | . 485324 | . 760789 | .343553 | . 458793 | . 488074 | .483596 |
|  | WYE VALLEY WEST | . 495797 | 7.491933 | . 371565 | . 386272 | . 522631 | . 486498 | . 488540 |
|  | RHIWARTH | . 496360 | -. 498855 | . 410573 | . 593281 | . 465604 | . 485306 | . 497010 |
|  | GORN | . 506520 | . 506512 | .730286 | . 629903 | . 540418 | . 486859 | . 509074 |
|  | LLANIDLOES | . 512633 | . 515325 | . 304213 | . 470246 | . 482887 | . 493318 | . 521443 |
|  | NANTY WEST | . 529454 | . 525717 | . 593288 | . 395237 | . 554384 | . 506195 | . 530418 |
|  | CABLAN | . 536848 | . 538108 | . 651555 | . 531049 | . 516053 | . 525823 | . 536053 |
|  | CEFNMAENLLWYD | . 586812 | 2.587191 | . 376429 | . 717169 | . 602591 | . 587485 | . 578299 |
|  | CRAIG RHIWARTH | . 588560 | . 590083 | . 757290 | . 642067 | . 585161 | . 603474 | . 580518 |
|  | ABERDAUNANT | . 598642 | . 593847 | . 584333 | . 478759 | . 586778 | . 619648 | . 592391 |
|  | TYISA | . 598840 | . 599128 | . 435681 | . 499655 | . 619470 | . 633361 | . 609141 |
|  | CYFARTHA | . 603517 | . 606571 | . 844629 | . 698025 | . 572173 | . 642655 | . 621543 |
|  | SNOW BROOR | . 610575 | . 616819 | . 459241 | . 760192 | . 665437 | . 646736 | . 624186 |
|  | LIANREAIADR | . 641138 | . 630516 | . 591405 | .572697 | . 568215 | . 646441 | . 621962 |
|  | MID-WALES | . 644380 | . 648309 | . 885150 | . 465246 | . 722998 | . 644197 | . 628350 |
|  | NANTMELYN | . 685623 | .680331 | . 380551 | . 693321 | . 595743 | . 655018 | . 664552 |
|  | CYMOROG | . 688285 | . 693553 | . 872858 | . 907574 | . 785500 | . 648888 | .697033 |
|  | PANTMAWR | . 699804 | . 706103 | . 781990 | . 746876 | .610606 | $.649488$ | $.729323$ |
|  | SEGLENLAS | . 718552 | . 717989 | . 396456 | . 480174 | . 812613 | . 660525 | . 743896 |
|  | CHMVRON | . 732042 | . 729219 | 1.130463 | . 624827 | . 640434 | . 683773 | . 735369 |
|  | van east | . 747418 | - 739801 | . 498129 | . 990304 | . 817397 | . 718344 | . 717435 |
|  | LLANGYNOG NORTE AND SOUTH | . 752822 | . 749742 | . 657505 | . 926589 | . 688852 | . 760517 | . 713860 |
|  | GLYN | . 754510 | . 759051 | 1.178673 | . 509465 | . 797807 | . 804191 | .739000 |
|  | WYE | . 760377 | - 767735 | . 228649 | . 510945 | . 756176 | . 841938 | . 783573 |
|  | CWMRICRET | . 777730 | - .775801 | 1.124013 | 1.007987 | . 755782 | . 866502 | . 819174 |
|  | BHLCH CREOLAN | . 785420 | -783258 | . 866766 | 1.125781 | . 838038 | . 872409 | . 820407 |
|  | FRONTBALLIAN | . 826521 | . 823003 | . 299633 | . 627064 | . 730631 | . 890537 | . 820607 |
|  | PENRALT | . 833128 | . 844337 | 1.557972 | . 413581 | . 974468 | . 872147 | . 798755 |
|  | FRONVELLAN | . 865911 | . 856258 | . 352564 | . 993179 | . 689515 | . 832659 | $.801017$ |
|  | BRYNYFEDWEN | . 876411 | . 873300 | . 836718 | 1.391328 | 1.073532 | . 796461 | . 855069 |
|  | SEVERN WATER | . 902058 | . 909996 | 1.539676 | . 857370 | . 681459 | . 789671 | . 949688 |
|  | SEVERN | . 983746 | .980882 | . 052573 | .381433 | 1.230470 | . 837798 | 1.051869 |
| DETERMINATION COEF. |  | 1.000000 | - . 998958 |  |  |  |  |  |
| MODEL ERROR METRICS |  | . 000000 | . 000136 | . 504105 | . 195727 | . 016252 | . 005112 | . 001354 |

## APPENDIX FOUR

Program POLYWORK.BAS

PROGRAM POLYWORK.BAS
A PROGRAM TO GENERATE A POLYNOMIAL REGRESSION EQUATION
OR USE A POLYNOMIAL EQUATION TO GENERATE A CURVE OR DISTRIBUTION
THE INPUT AND OUTPUT DATA ARE STORED IN COMMA SEPARATED VARIABLE
FILES AMENABLE TO SPREADSHEET MANIPULATION.
THESE FILES BEAR THE EXTENSIONS SHOWN:-

| REGRESSION INPUT DATA | . CSV |
| :--- | :--- |
| POLYNOMIAL REGRESSION COEFFICIENTS | . COF |
| FITTED CURVE CO-ORDINATES | .CUR |
| FREQUENCY |  |

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21 MAY 1995

THIS PROGRAM IS WRITTEN IN MICROSOFT QBASIC
' VARIABLE TYPE DEFAULTS
DEFDBL A-H, O-R, T-Z
DEFSTR S
DEFINT I-K, M-N
DEFLNG L

- SEGMENT DECLARATIONS

DECLARE SUB CSVIN (ID, SF)
DECLARE SUB COEFOUT (IV, CD, CC, CE)
DECLARE SUB COFIN (SF, IN, DB, UB)
DECLARE SUB DATATRANS (ISW, IU, IV, N, $X(), Y())$
DECLARE SUB DEFINITE (ID, FD, VM, DB, UB, C(), A())
DECLARE SUB DISIN (SF, IP, DB, UB, JN)
DECLARE SUB GETTER ()
DECLARE SUB HEADERIN (IU, SF, SHT, SST, SXT, SYT)
DECLARE SUB HEADEROUT (IV, SF, SHT, SST, SXT, SYT)
DECLARE SUB INTEGRALOUT (IV, FD, VM)
DECLARE SUB MENU (IC)
DECLARE SUB MOMENTSOUT (IV, DM, DS, ZT)
DECLARE SUB NOTE (S)
DECLARE SUB PICK (KT, I, L, R, $S, I Y, ~ I X, ~ M K, ~ S F)$
DECLARE SUB PLACE (KT, I, L, R, S, IY, IX, IU, MK, JF, SF, IB)
DECLARE SUB POLYDIST (ID, IP, JN, N, DB, UB, DM, DS, ZT, C(), A(), X(), Y())
DECLARE SUB POLYREG (ID, $N, X(), Y(), C(), A(), C D, C C, C E)$
DECLARE SUB POLYSIMP (ID, FS, N, X(), Y())
DECLARE SUB POLYSYN (ID, N, IN, DB, UB, C(), A(), X(), Y())
DECLARE SUB RDUMP (SPATH, R(), ID, J)
DECLARE SUB SIMPSON (FS, N, X(), Y())

- COMMON VARIABLES

COMMON SHARED IA, SA
COMMON SHARED IZ, IERR
COMMON SHARED SC, SM
COMMON SHARED S6, S8, S9, S10, S14, S15
COMMON SHARED SE, SD, SN, SW, SS, SO, SL, SU
COMMON SHARED SA8, SI1, SI2, SI4, SF13.6
' STATIC ARRAY DEFINITIONS
DIM C(12), A(12), X(1000), Y(1000)
' DYNAMIC ARRAY DEFINITIONS
' ( none )
' DEVICE ATTRIBUTIONS
SCREEN 12: WINDOW (1, 1)-(640, 480)
' LOGICAL UNIT AND PATHNAME SETTINGS
$I U=1: I V=2: S P=$ "C:\QBASIC QQBFILES $\backslash "$
' FORMAT DEFINITIONS
SI1 = "\#": SI2 = "\#\#": SI4 = "\#\#\#\#": SF13.6 = "\#\#\#\#\#\#.\#\#\#\#\#\#"
' NUMERICAL CONSTANT DEFINITIONS
( none)
STRING CONSTANT DEFINITIONS
SC = ": ": SM = ","
S6 = SPACE\$ (6) : S8 = SPACE\$ (8) : S9 = SPACE\$ (9): S10 = SPACE\$ (10)
S14 = SPACE\$ (14): S15 = SPACE\$ (15)
' TEXT VARIABLE DEFINITIONS
SE = "ENTER THE ": SD = "DEGREE OF ": SN = "NUMBER OF ": SW = "FILENAME ( WITHOUT
EXTENSION )"
SO = "ORDINAL ": SS = "ABSCISSAL ": SL = "LOWER BOUND ": SU = "UPPER BOUND "

```
' ** THE ALGORITHM **
DO
            IERR = 0
            ERASE C, A, X, Y
            MENU IC
            SELECT CASE IC
            CASE 1
                CSVIN ID, SF
                OPEN "I", IU, SP + SF + ".CSV"
                OPEN "O", IV, SP + SF + ".COF"
                HEADERIN IU, SF, SHT, SST, SXT, SYT
                DATATRANS 1, IU, IV, N, X(), Y()
                IF ID < 13 AND N - ID > 1 THEN
                    POLYREG ID, N, X(), Y(), C(), A(), CD, CC, CE
                    ELSE
                    CLS : COLOR 4: PRINT "INSUFFICIENT DATA"
                    PRINT "OR NUMBER OF DEGREES EXCEEDS 12"
                            IERR = 1
                    BEEP: SLEEP 2
                END IF
                IF IZ = 1 AND IERR = 0 THEN
                    HEADEROUT IV, SF, SHT, SST, SXT, SYT
                            DATATRANS 2, IU, IV, ID + 1, C(), A()
                            COEFOUT IV, CD, CC, CE
                    ELSE
                    IF IZ = O THEN
                            CLS : COLOR 4: PRINT "NO UNIQUE SOLUTION"
                            IERR = 1
                            BEEP: SLEEP 2
                    END IF
                END IF
                CLOSE IU, IV
        CASE 2
                COFIN SF, IN, DB, UB
                OPEN "I", IU, SP + SF + ".COF"
                OPEN "O", IV, SP + SF + ".CUR"
                HEADERIN'IU,' SF, SHT, SST, SXT, SYT
                DATATRANS 1, IU, IV, IDD, C(), A()
                ID = IDD - 1
                POLYSYN ID, N, IN, DB, UB, C(), A(), X(), Y()
                DEFINITE ID, FD, VM, DB, UB, C(), A()
                HEADEROUT IV, SF, SHT, SST, SXT, SYT
                DATATRANS 2, IU, IV, N, X(), Y()
                INTEGRALOUT IV, FD, VM
                CLOSE IU, IV
            CASE 3
                DISIN SF, IP, DB, UB, JN
                OPEN "I", IU, SP + SF + ".COF"
                OPEN "O", IV, SP + SF + ".DIS"
                HEADERIN IU, SF, SHT, SST, SXT, SYT
                DATATRANS 1, IU, IV, IDD, C(), A()
                ID = IDD - 1
                POLYDIST ID, IP, JN, N, DB, UB, DM, DS, ZT, C(), A(), X(), Y()
                HEADEROUT IV, SF, SHT, SST, SXT, SYT
                DATATRANS 2, IU, IV, N, X(), Y()
                MOMENTSOUT IV, DM, DS, ZT
                CLOSE IU, IV
            END SELECT
                LOOP UNTIL IC = 4
                CLS
                END
SUB COEFOUT (IV, CD, CC, CE)
A SUBROUTINE TO WRITE THE POLYNOMIAL REGRESSION SUMMARY CORRELATIVE METRICS
ARGUMENTS:
    IV THE OUTPUT LOGICAL UNIT NUMBER
    CD THE COEFFICIENT OF DETERMINATION
    CC THE COEFFICIENT OF CORRELATION
    CE THE STANDARD ERROR OF THE ESTIMATE
( EMPLOYED CLICHES ARE COMMON SHARED )
PRINT #IV, "COEFFICIENT OF DETERMINATION = "; SM; CD
PRINT #IV, "COEFFICIENT OF CORRELATION = "; SM; CC
PRINT #IV, "STANDARD ERROR OF THE ESTIMATE = "; SM; CE
END SUB
SUB COFIN (SF, IN, DB, UB)
' A SUBROUTINE TO OBTAIN KEYBOARD DATA FOR SEGMENT POLYSYN
ARGUMENTS:
            SF THE GENERIC FILENAME
            IN THE NUMBER OF ABSCISSAL INTERVALS
```

```
            DB THE ABSCISSAL LOWER BOUND
            UB THE ABSCISSAL UPPER BOUND
( EMPLOYED CLICHES ARE COMMON SHARED )
    CLS : COLOR 2
    LOCATE 8, 33: PRINT "SEGMENT POLYSYN"
    LOCATE 10, 19: PRINT "A Segment to Synthesise The Curve Expressing"
    LOCATE 11, 13: PRINT "The Polynomial Equation Defined by Coefficients in *.COF"
    LOCATE 12, 18: PRINT "And to Record the Curve Co-ordinates in *.CUR"
    LOCATE 14, 14: PRINT SE + SW + SC
    LOCATE 15, 14: PRINT SE + SN + SS + "INTERVALS " + SC
    LOCATE 16, 14: PRINT SE + SS + SL + S8 + SC
    LOCATE 17, 14: PRINT SE + SS + SU + S8 + SC
    PICK 4, 0, 0, 0, SF, 14, 56, 3, SA8
    PICK 1, IN, 0, 0, S, 15, 56, 3, SI4
    PICK 3, 0, 0, DB, S, 16, 56, 3, SF13.6
    PICK 3, 0, 0, UB, S, 17, 56, 3, SF13.6
    END SUB
    SUB CSVIN (ID, SF)
    ' A SUBROUTINE TO OBTAIN KEYBOARD DATA FOR SEGMENT POLYREG
    ARGUMENTS:
        ID THE DEGREE OF THE POLYNOMIAL TO BE FITTED
        SF THE GENERIC FILENAME
    ( EMPLOYED CLICHES ARE COMMON SHARED )
        CLS : COLOR 2
        LOCATE 10, 33: PRINT "SEGMENT POLYREG"
        LOCATE 12, 12: PRINT "A Segment to Perform an ID-degree Polynomial Regression"
        LOCATE 13, 17: PRINT "On a Curve Defined by Co-ordinates in File *.CSV"
        LOCATE 15, 17: PRINT SE + SD + "THE FITTED EQUATION " + SC
        LOCATE 16, 17: PRINT SE + SW + SC
        PICK 1, ID, 0, 0, S, 15, 59, 3, SI2
        PICK 4, 0, 0, 0, SF, 16, 59, 3, SA8
        END SUB
        SUB DATATRANS (ISW, IU, IV, N, X(), Y())
    ' A SUBROUTINE TO READ OR WRITE THE CO-ORDINATES DATA LIST
        ARGUMENTS:
            ISW THE TRANSPUT SELECTOR SWITCH
                1 READ
                    2 WRITE
            IU THE INPUT LOGICAL UNIT NUMBER
            IV THE OUTPUT LOCICAL UNIT NUMBER
            N THE NUMBER OF DATA CO-ORDINATES
            X() THE ARRAY OF ABSCISSAL CO-ORDINATES
            Y() THE ARRAY OF ORDINAL CO-ORDINATES
( EMPLOYED CLICHES ARE COMMON SHARED )
    IF ISW = 1 THEN
                INPUT #IU, N
                FOR I = 1 TO N: INPUT #IU, X(I), Y(I): NEXT I
            ELSE
                PRINT #IV, N
                    FOR I = 1 TO N: PRINT #IV, X(I); SM; Y(I): NEXT I
            END IF
            END SUB
            SUB DEFINITE (ID, FD, VM, DB, UB, C(), A())
' A SUBROUTINE TO COMPUTE THE DEFINITE INTEGRAL OF
' A POLYNOMIAL EQUATION BETWEEN DB AND UB
' AND TO ESTABLISH THE INTEGRAL MEAN VALUE BETWEEN
' THOSE BOUNDS
    ARGUMENTS :
    ID THE DEGREE OF THE POLYNOMIAL
            FD THE DEFINITE INTEGRAL
            VM THE INTEGRAL MEAN VALUE
            DB THE INTERVAL LOWER BOUND
            UB THE INTERVAL UPPER BOUND
            C() THE ARRAY OF TERM EXPONENTS
            A() THE ARRAY OF TERM COEFFICIENTS
    FOR I = 1 TO ID + 1
        T1 = T1 + (A(I) * DB ^ I) / I: T2 = T2 + (A(I) * UB ^ I) / I
    NEXT I
    FD = T2 - T1: VM = FD / (UB - DB)
    END SUB
    SUB DISIN (SF, IP, DB, UB, JN)
' A SUBROUTINE TO OBTAIN KEYBOARD DATA FOR SEGMENT POLYDIST
    ARGUMENTS:
        SF THE GENERIC FILENAME
```

```
    IP THE NUMBER OF SYNTHETIC ABSCISSAL POINTS
    DB THE ABSCISSAL LOWER BOUND
    UB THE ABSCISSAL UPPER BOUND
    JN THE NUMBER OF ORDINAL INCREMENTS
( EMPLOYED CLICHES ARE COMMON SHARED )
CLS : COLOR 2: LOCATE 6, 33: PRINT "SEGMENT POLYDIST"
LOCATE 8, 14: PRINT "A Segment to Generate a Grouped Frequency Distribution"
LOCATE 9, 7: PRINT "From Log(Quantity) vs. Rank Data Expressed by a Polynomial
Equation"
    LOCATE 10, 22: PRINT "Whose Co-efficients are Read from *.COF"
    LOCATE 11, 23: PRINT "The Distribution is Recorded in *.DIS"
    LOCATE 13, 10: PRINT SE + SW + S6 + SC
    LOCATE 14, 10: PRINT SE + SN + "SYNTHETIC " + SS + "POINTS" + SC
    LOCATE 15, 10: PRINT SE + SS + SL + S14 + SC
    LOCATE 16, 10: PRINT SE + SS + SU + S14 + SC
    LOCATE 17, 10: PRINT SE + SN + SO + "INCREMENTS" + S8 + SC
    PICK 4, 0, 0, 0, SF, 13, 58, 3, SA8
    PICK 1, IP, 0, 0, S, 14, 58, 3, SI4
    PICK 3, 0, 0, DB, S, 15, 58, 3', SF13.6
    PICK 3, 0, 0, UB, S, 16, 58, 3, SF13.6
    PICK 1, JN, 0, 0, S, 17, 58, 3, SI2
    END SUB
    SUB GETTER
A SUBROUTINE TO ACCEPT A KEYSTROKE AS SA AND TO YIELD ITS ASCII CODE AS IA
( THE ARGUMENTS SA AND IA ARE COMMON SHARED )
    DO
        SA = INKEY$
    LOOP UNTIL SA <> ""
    IA = ASC (SA)
    END SUB
    SUB HEADERIN (IU, SF, SHT, SST, SXT, SYT)
A SUBROUTINE TO READPOLYNOMIAL FILE HEADER DATA
    ARGUMENTS:
            IU THE INPUT FILE LOGICAL UNIT NUMBER
            SF THE GENERIC FILE NAME
            SHT THE FILE MAIN HEADING
            SST THE FILE SUB HEADING
            SXT THE ABSCISSAL DESCRIPTOR
            SYT THE ORDINAL DESCRIPTOR
            INPUT #IU, SF
            INPUT #IU, SHT
            INPUT #IU, SST
            INPUT #IU, SXT
            INPUT #IU, SYT
    END SUB
    SUB HEADEROUT (IV, SF, SHT, SST, SXT, SYT)
A SUBROUTINE TO WRITE POLYNOMIAL FILE HEADER DATA
    ARGUMENTS:
            IV THE OUTPUT FILE LOGICAL UNIT NUMBER
            SF THE GENERIC FILENAME
            SHT THE FILE MAIN HEADING
            SST THE FILE SUB HEADING
            SXT THE ABSCISSAL DESCRIPTOR
            SYT THE ORDINAL DESCRIPTOR
            PRINT #IV, SF
            PRINT #IV, SHT
            PRINT #IV, SST
            PRINT #IV, SXT
            PRINT #IV, SYT
            END SUB
                            SUB INTEGRALOUT (IV, FD, VM)
' A SUBROUTINE TO WRITE THE DEFINITE INTEGRAL
' AN THE INTEGRAL MEAN VALUE
    ARGUMENTS:
            IV THE OUTPUT LOGICAL UNIT NUMBER
            FD THE DEFINITE INTEGRAL
            VM THE INTEGRAL MEAN VALUE
( EMPLOYED CLICHES ARE COMMON SHARED )
    PRINT #IV, "DEFINITE INTEGRAL = "; SM; FD
    PRINT #IV, "MEAN VALUE = "; SM; VM
    END SUB
    SUB MENU (IC)
```

```
A SUBROUTINE TO OBTAIN THE FUNCTION CHOICE FROM THE USER
        ARGUMENT :
IC THE USER FUNCTION CHOICE
( EMPLOYED CLICHES ARE COMMON SHARED )
        CLS : COLOR 2
        LOCATE 8, 31: PRINT "PROGRAM POLYWORK.BAS"
        LOCATE 10, 14: PRINT "A Program to Generate a Polynomial Regression Equation"
        LOCATE 11, 9: PRINT "Or Use a Polynomial Equation to Generate a Curve or Distribution"
        LOCATE 13, 3: PRINT "1 Generate a Polynomial Regression Equation"
        LOCATE 14, 3: PRINT "2 Generate XY-Plot Data from Polynomial Coefficients"
        LOCATE 15, 3: PRINT "3 Generate a Grouped Frequency Distribution using Polynomial
Coefficients"
    LOCATE 16, 3: PRINT "4 Quit"
        LOCATE 18, 27: PRINT "Select Your Option by Number"
        GETTER
        IC = IA - 48
        END SUB
        SUB MOMENTSOUT (IV, DM, DS, ZT)
' A SUBROUTINE TO WRITE THE FREQUENCY DISTRIBUTION SUMMARY METRICS
        ARGUMENTS:
            IV THE OUTPUT LOGICAL UNIT NUMBER
            DM THE MEAN OF THE DISTRIBUTION
            DS THE STANDARD DEVIATION OF THE DISTRIBUTION
            ZT THE SUM OF THE ORDINAL QUANTITIES
    ( EMPLOYED CLICHES ARE COMMON SHARED )
        PRINT #IV, "DISTRIBUTION MEAN = "; SM; DM
        PRINT #IV, "DISTRIBUTION STANDARD DEVIATION = "; SM; DS
        PRINT #IV, "SUM OF ORDINAL QUANTITIES = "; SM; ZT
        END SUB
        SUB NOTE (S)
A SUBROUTINE TO SOUND A NOTE UPON THE COMPUTER SPEAKER
    ARGUMENT:
            S THE NOTE SPECIFIER STRING "IN$NL" e.g. "2B0506"
                I THE OCTAVE NUMBER ( 0-6)
                N$ THE NOTE LETTER ( ABCDEFG )
                N THE NOTE NUMBER ( 0-84 )
                L THE LENGTH OF THE NOTE ( 1-64)
    I = VAL (MID$ (S, 1, 1)): N$ = MID$ (S, 2, 1)
    N = VAL (MID$(S, 3, 2)): L = VAL (MID$ (S, 5, 2))
    PLAY "O" + STR$(I) + "N" + STR$(N) + "L" + STR$(L) + "X" + VARPTR$(N$)
    END SUB
    SUB PICK (KT, I, L, R, S, IY, IX, MK, SF)
    A SUBROUTINE TO OBTAIN A VARIABLE OF TYPE KT AT SCREEN POSITION IY,IX
    ARGUMENTS:
        KT THE DATUM TYPE CHOICE
                SHORT INTEGER
                LONG INTEGER
                DOUBLE PRECISION REAL
                STRING
                THE SHORT INTEGER
                THE LONG INTEGER TO BE OBTAINED ( OPTION )
                THE REAL TO BE OBTAINED ( OPTION )
                THE STRING TO BE OBTAINED ( OPTION )
                THE STARTING SCREEN ROW
                THE STARTING SCREEN COLUMN
                THE PRINTING COLOR
                THE PROPER ( OR DEFAULT ) PRINTING FORMAT
                    ( "SAnn" TRUNCATES A STRING TO nn CHARACTERS )
    SDEL = CHR$ (0) + CHR$ (83)
    COLOR MK: LOCATE IY, IX: IO = I: LO = L: RO = R: SOL = S: L1 = LEN(SF)
    SELECT CASE KT
        CASE 1
                IF LEN(STR$(I)) > L1 THEN L1 = LEN(STR$(I))
        CASE 2
            IF LEN(STR$(L)) > L1 THEN L1 = LEN(STR$(L))
        CASE 3
                IF LEN(STR$(R)) > L1 THEN L1 = LEN(STR$(R))
        CASE 4
                L1 = LEN(S)
    END SELECT
    PRINT SPACE$ (L1)
    DO
        LOCATE IY, IX: PRINT "."
        FOR II = 1 TO 10: NEXT II
        LOCATE IY, IX: PRINT " "
```

```
    SA = INKEY$
    LOOP UNTIL SA <> ""
    IA = ASC (SA)
    IF IA <> 13 THEN
    SCON = "": IT = IX: LOCATE IY, IX
        DO
            IF SA = SDEL THEN
                        IT = IT - I
                        LOCATE IY, IT: PRINT SPACE$(1)
                        LOCATE IY, IT: SCON = LEFT$(SCON, LEN(SCON) - 1)
                ELSE
                    SELECT CASE KT
                                CASE 1 TO 3
                                IF IA > 47 AND IA < 58 OR IA = 46 OR IA = 45 THEN
                                    SCON = SCON + SA
                                    PRINT SA;
                                    IT = IT + I
                                    ELSE
                                    IF IA <> 13 THEN NOTE "2B0506"
                                    END IF
                                    CASE 4
                                    IF IA > 31 AND IA < 127 THEN
                                    SCON = SCON + SA
                                    PRINT SA;
                                    IT = IT + 1
                                    ELSE
                                    IF IA <> 13 THEN NOTE "2B0506"
                                    END IF
                    END SELECT
            END IF
            GETTER
        LOOP UNTIL IA = 13
        SELECT CASE KT
            CASE 1
                I = INT(VAL(SCON) + . 5)
            CASE 2
                L = INT (VAL (SCON) + .5)
            CASE 3
                R = VAL (SCON)
        END SELECT
    ELSE
        I = IO: L = LO: R = RO
    END IF
L1 = LEN(SF): IF LEN(SCON) > L1 THEN L1 = LEN(SCON): PRINT SPACE$(L1)
LOCATE IY, IX: PRINT SPACE$(L1): LOCATE IY, IX
SELECT CASE KT
    CASE 1
        PRINT USING SF; I
    CASE 2
    PRINT USING SF; I
    CASE }
        PRINT USING SF; R
    CASE 4
        LOCATE IY, IX: PRINT SPACE$(LEN(SCON)) : LOCATE IY, IX
        IF SOL <> "" AND SCON = "" THEN SCON = SOL
        IF LEFT$(SF, 2) = "SA" THEN
            S = LEFT$ (SCON, VAL (MID$ (SF, 3)))
            ELSE
                S = SCON
        END IF
        PRINT S
END SELECT
END SUB
SUB PLACE (KT, I, L, R, S, IY, IX, IU, MK, JF, SF, IB)
A SUBROUTINE TO PLACE A VARIABLE OF TYPE KT AT SCREEN POSITION IY, IX
OR ALTERNATIVELY PLACE THE VARIABLE WITHIN A PADDED REPORT FILE
    ARGUMENTS:
        KT THE DATUM TYPE CHOICE
                SHORT INTEGER
                LONG INTEGER
                DOUBLE PRECISION REAL
                STRING
    THE SHORT INTEGER TO BE PRINTED (OPTION )
    THE LONG INTEGER TO BE PRINTED ( OPTION )
    THE REAL TO BE PRINTED ( OPTION)
    THE STRING TO BE PRINTED ( OPTION )
    THE STARTING SCREEN ROW
    THE STARTING SCREEN COLUMN
    THE LOGICAL UNIT NUMBER
        1 PRINT TO THE SCREEN
        2 PRINT TO A REPORT FILE
```

```
    MK THE NOMINAL PRINTING COLOR
    JF THE LINE FEED SUPPRESSOR SWITCH
        O FOLLOW WITH A LINE FEED
        1 DO NOT FOLLOW WITH A LINE FEED
        THE REQUIRED PRINTING FORMAT
        THE NUMBER OF FORWARD PADDING SPACES
    IF SF = "" THEN SF = "#########"
    COLOR MK
    SELECT CASE IU
    CASE 1
        LOCATE IY, IX
            SELECT CASE KT
                    CASE 1
                        PRINT USING SF; I
                    CASE 2
                        PRINT USING SF; L
                    CASE 3
                                    PRINT USING SF; R
                    CASE 4
                PRINT S
                END SELECT
    CASE 2
        IF IB > 0 THEN PRINT #IU, SPACE$(IB);
        IF JF = 1 THEN SCC = ";" ELSE SCC = CHR$(13) + CHR$(10)
            SELECT CASE KT
            CASE 1
                PRINT #IU, USING SF; I; SCC
            CASE 2
                PRINT #IU, USING SF; L; SCC
            CASE 3
                    PRINT #IU, USING SF; R; SCC
                    CASE 4
                    PRINT #IU, S
                END SELECT
    IF JF = 0 THEN PRINT #LU,
    END SELECT
    END SUB
    SUB POLYDIST (ID, IP, JN, N, DB, UB, DM, DS, ZT, C(), A(), X(), Y())
' A SUBROUTINE TO GENERATE A GROUPED FREQUENCY DISTRIBUTION
FROM DATA EXPRESSED BY A POLYNOMIAL EQUATION
    ARGUMENTS:
    ID THE DEGREE OF THE REGRESSION POLYNOMIAL
    IP THE NUMBER OF SYNTHETIC ABSCISSAL POINTS
    JN THE NUMBER OF ORDINAL INCREMENTS
    N THE NUMBER OF DATA PAIRS
    DB THE ABSCISSAL LOWER BOUND
    UB THE ABSCISSAL UPPER BOUND
    DM THE MEAN OF THE DISTRIBUTION
    DS THE STANDARD DEVIATION OF THE DITRIBUTION
    ZT THE SUM OF THE ORDINAL QUANTITIES
    C() THE ARRAY OF TERM EXPONENTS
    A() THE ARRAY OF TERM COEFFICIENTS
    X() THE ARRAY OF ABSCISSAL CO-ORDINATES
    Y() THE ARRAY OF ORDINAL CO-ORDINATES
    DIM Z(5000)
    IF IP > 5000 THEN IP = 5000
    N}=\textrm{JN}+1:R=\textrm{UB}-\textrm{DB}: ZT=0
    ZX = -9.999999999999999D+33: ZN = 9.999999999999999D+33
    COLOR 13
    LOCATE 19, 29: PRINT "POINTS DEFINED :"
    LOCATE 20, 29: PRINT "POINTS ALLOCATED:"
    COLOR 4
    XI = R / (IP - 1): XX = DB - XI
    FOR I = 1 TO IP
        XX = XX + XI
        FOR J = 1 TO ID + 1: Z(I) = Z(I) + A(J) * XX ^ C(J): NEXT J
        Z(I) = EXP(Z(I))
        IF Z(I) < ZN THEN ZN = Z(I)
        IF Z(I) > ZX THEN ZX = Z(I)
    ZT}=\textrm{ZT}+\textrm{Z}(I
    LOCATE 19, 47: PRINT I
    NEXT I
    RY = ZX - ZN: YI = RY / JN
    X(1) = ZN: FOR I = 2 TO N: X(I) = X(I - I) + YI: NEXT I
    COLOR 6
    FOR I = 1 TO IP
        LOCATE 22, 1: J = INT((Z(I) - ZN) / YI) + 1: Y(J) = Y(J) + 1
        LOCATE 20, 47: PRINT I
    NEXT I
```

```
\(\mathrm{TF}=0!: \mathrm{TFX}=0!: \mathrm{TFX} 2=0\) !
```

FOR $I=1$ TO JN
$X M=X(I)+Y I / 2$
$\mathrm{TF}=\mathrm{TF}+\mathrm{Y}(\mathrm{I}): \mathrm{TFX}=\mathrm{TFX}+\mathrm{Y}(\mathrm{I})$ * XM
$\mathrm{TFX} 2=\mathrm{TFX} 2+\mathrm{Y}(\mathrm{I}) * \mathrm{XM} \wedge 2$
NEXT I
$\mathrm{DM}=\mathrm{TFX} / \mathrm{TF}: \mathrm{DS}=\mathrm{SQR}(\mathrm{TFX} 2 / \mathrm{TF}-\mathrm{DM} * \mathrm{DM})$
END SUB
SUB POLYREG (ID, N, X(), Y(), C(), A(), CD, CC, CE)
' A SUBROUTINE TO PERFORM A POLYNOMIAL REGRESSION
ARGUMENTS:
ID THE DEGREE OF THE REGRESSION POLYNOMIAL
N THE NUMBER OF DATA CO-ORDINATES
X()$\quad$ THE ARRAY OF ABSCISSAL CO-ORDINATES
Y() THE ARRAY OF ORDINAL CO-ORDINATES
C() THE ARRAY OF TERM EXPONENTS
A() THE ARRAY OF TERM CO-EFFICIENTS
CD THE COEFFICIENT OF DETERMINATION
CC THE COEFFICIENT OF CORRELATION
CE THE STANDARD ERROR OF THE ESTIMATE
( IZ AND IERR ARE COMMON SHARED )
DIM AA (25), R(13, 14), T(14)
$K D=I D+1: K K D=I D+2$
' POPULATE THE SOLUTION MATRICES WITH A SYSTEM OF EQUATIONS
AA (1) $=N$
FOR I = 1 TO N
FOR $J=2$ TO 2 * ID $+1:$ AA $(J)=A A(J)+X(I) \wedge(J-1):$ NEXT $J$
FOR K = 1 TO KD
$R(K, K K D)=T(K)+Y(I) * X(I) \wedge(K-1): T(K)=R(K, K K D)$
NEXT K
$T(K K D)=T(K K D)+Y(I) \wedge 2$
NEXT I
' SOLVE THE SYSTEM OF EQUATIONS IN THE MATRICES
FOR J = 1 TO KD: FOR K = 1 TO KD: R(J, K) = AA (J + K - 1): NEXT K: NEXT J
$\mathrm{J}=0$
DO
$J=J+1: K=J$
IZ $=0: \operatorname{IERR}=1$
DO
IF $R(K, J)$ <> 0 THEN
FOR $I=1$ TO KKD: RS $=R(J, I): R(J, I)=R(K, I): R(K, I)=R S: N E X T I$
$Z=1 / R(J, J): F O R I=1$ TO KKD: R(J, I) $=Z * R(J, I): N E X T I$
FOR KK = 1 TO J - 1
$Z=-R(K K, J): F O R I=1$ TO KKD: $R(K K, I)=R(K K, I)+Z * R(J, I): N E X T$
I
NEXT KK
FOR KK = J + 1 TO KD
$Z=-R(K K, J): F O R I=1$ TO KKD: R(KK, I) $=R(K K, I)+Z * R(J, I): N E X T$
I
NEXT KK
IZ = 1: IERR = 0
END IF
$K=K+1$
LOOP UNTIL $\mathrm{K}=\mathrm{KD}$ OR IZ $=1$
LOOP UNTIL $\mathrm{J}=\mathrm{KD}$ OR $\operatorname{IERR}=1$
' LOAD THE RESULTS ARRAYS
FOR I = 1 TO KD
$C(I)=I-1: A(I)=R(I, K K D)$
NEXT I

- COMPUTE THE REGRESSION ANALYSIS
IF IERR = 0 THEN
$P=0: F O R J=2 T O K D: P=P+R(J, K K D) *(T(J)-A A(J) * T(1) / N): N E X T J$
$\mathrm{P}=0: \mathrm{FOR} \mathrm{J}=2 \mathrm{TO}$ KD: $\mathrm{P}=\mathrm{P}+\mathrm{R}(\mathrm{J}, \mathrm{KKD}) *$ * (T) J$)$
$\mathrm{Q}=\mathrm{T}(\mathrm{KKD})-\mathrm{T}(1) \wedge 2 / \mathrm{N}: \mathrm{Z}=\mathrm{Q}-\mathrm{P}: \mathrm{I}=\mathrm{N}-\mathrm{KD}$
$Q=T(K K D)-T(1) \wedge 2 /$
$C D=P / Q: C C=S Q R(C D)$
$\mathrm{CD}=\mathrm{P} / \mathrm{Q}: \mathrm{CC}=\mathrm{SQR}$
IF $\mathrm{Z}<0$ THEN $Z=0$
$\mathrm{CE}=\operatorname{SQR}(\mathrm{Z} / \mathrm{I})$
END IF
END SUB
SUB POLYSIMP (ID, FS, N, X(), Y())
' A SUBROUTINE TO PERFORM A SIMPSON'S RULE INTEGRATION
' A SUBROUTINE TO PERFORM A SIMPSON'S RULE IN
' IF THE POLYNOMIAL DEGREE IS LESS THAN FOUR
, THE SHORT FORM OF THE INTEGRATING ALGORITHM IS EMPLOYED.
ARGUMENTS:
ID THE DEGREE OF THE REGRESSION POLYNOMIAL
FS THE SIMPSONIAN INTEGRAL
N THE NUMBER OF DATA CO-ORDINATES
X() THE ARRAY OF ABSCISSAL CO-ORDINATES
$Y() \quad$ THE ARRAY OF ORDINAL CO-ORDINATES

```
    IF ID < 4 THEN
                FS = ((X(N) - X(1)) / 6) * (Y(1) + 4 * Y(INT (N / 2) + 1) + Y(N))
            ELSE
                SIMPSON FS, N, X(), Y()
    END IF
    END SUB
    SUB POLYSYN (ID, N, IN, DB, UB, C(), A(), X(), Y())
A SUBROUTINE TO SYNTHESISE THE CURVE EXPRESSING A POLYNOMIAL EQUATION
    ARGUMENTS:
        ID THE DEGREE OF THE REGRESSION POLYNOMIAL
        N THE NUMBER OF DATA CO-ORDINATES
        IN THE NUMBER OF ABSCISSAL INTERVALS
        DB THE ABSCISSAL LOWER BOUND
            THE ABSCISSAL UPPER BOUND
            THE ARRAY OF TERM EXPONENTS
            THE ARRAY OF TERM CO-EFFICIENTS
            THE ARRAY OF ABSCISSAL CO-ORDINATES
            THE ARRAY OF ORDINAL CO-ORDINATES
    N = IN + 1: AI = (UB - DB) / IN: X(0) = DB - AI
    FOR I = 1 TO N
        X(I) = X(I - 1) + AI: Y(I) = 0!
        FOR J = 1 TO ID + 1
            Y(I) = Y(I) + A(J) * X(I) ^ (J - 1)
        NEXT J
    NEXT I
    END SUB
    SUB RDUMP (SPATH, R(), ID, J)
    OPEN "A", 4, SPATH
    PRINT #4, "J = "; J
    PRINT #4,
    FOR JJ = 1 TO ID + 2
    FOR KK = 1 TO ID + 2
    PRINT #4, USING "######.######"; R(JJ, KK);
    PRINT #4, " ";
    NEXT KK
    PRINT #4,
    NEXT JJ
    CLOSE 4
    END SUB
    SUB SIMPSON (FS, N, X(), Y())
' A SUBROUTINE TO PERFORM A SIMPSON'S RULE INTEGRATION
UPON THE CURVE COMPUTED BY SEGMENT POLYSYN
    ARGUMENTS:
        FS THE SIMPSONIAN INTEGRAL
        N THE NUMBER OF DATA CO-ORDINATES
        X() THE ARRAY OF ABSCISSAL CO-ORDINATES
        Y() THE ARRAY OF ORDINAL CO-ORDINATES
    IN = N - 1
    IF IN MOD 2 = 0 THEN
        J = -2: K = 2: T = 0!
        FOR I = 2 TO IN
            J = -J: K = K + J: T = T + K * Y(I)
        NEXT I
        FS = ((X(N) - X(1)) / (3 * IN)) * (Y(1) + Y(N) + T)
        ELSE
            CLS : COLOR 4
            PRINT "ODD NUMBER OF ABSCISSAL INTERVALS:"
            PRINT "SIMPSONIAN INTEGRAL SET TO ZERO"
            IERR = 1
            FS = 0
            BEEP: SLEEP 2
    END IF
    END SUB
```


## APPENDIX FIVE

Program MINEMODL.BAS

PROGRAM MINEMODL.BAS
A PROGRAM TO GENERATE A SERIES OF FOURIER-POLYNOMIAL MODEL FITMENTS
TO LEAD MINE LOG PRODUCTION VERSUS RANK DATA AND TO OUTPUT THESE FITMENTS AS A COMPARATIVE TABLE.
THE INPUT AND OUTPUT DATA ARE STORED IN COMMA SEPARATED VARIABLE
FILES AMENABLE TO SPREADSHEET MANIPULATION.
THESE FILES BEAR THE EXTENSIONS SHOWN:-

| ORIGINAL RANKED DATA | . CSV |
| :--- | :--- |
| POLYNOMIAL REGRESSION COEFFICIENTS | . COF |
| MODEL FITMENTS REPORT | .MOD |

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THIS PROGRAM IS WRITTEN IN MICROSOFT QBASIC
' VARIABLE TYPE DEFAULTS
DEFDBL $A-H, O-R, T-Z$
DEFSTR S
DEFINT I-K, M-N
DEFLNG L
' SEGMENT DECLARATIONS
DECLARE SUB DATATRANS (ISW, IU, IV, N, X(), Y())
DECLARE SUB DETERMINATION (M, JC, JF, JE, W(), CD())
DECLARE SUB ERRORMETRIC (M, JC, JF, JE, W(), ER())
DECLARE SUB ERRORSET (SER())
DECLARE SUB ERRORSHOW (I, IC, SER())
DECLARE SUB FOURSERIES (M, M1, M2, N, X(), Y())
DECLARE SUB GETTER ()
DECLARE SUB HEADERIN (IU, SF, SHT, SST, SXT, SYT)
DECLARE SUB INSCREEN (SFO, SHT, SST, K, NM, NX, SFN(), IR())
DECLARE SUB LINEBLANKER (I, J)
DECLARE SUB META (M, SM)
DECLARE SUB MODELCALC (IU, K, M, NM, NX, SFO, SFN(), IR(), W(), CD(), ER(), SMN())
DECLARE SUB MODELRPT (ISW, IV, SFO, SFN(), SHT, SST, K, M, IR(), NM, NX, W(), CD(),
ER(), $\operatorname{SMN}())$
DECLARE SUB NOSEG (SK)
DECLARE SUB NOTE (S)
DECLARE SUB PICK (KT, I, L, R, S, IY, IX, MK, SF)
DECLARE SUB PLACE (KT, I, L, R, S, IY, IX, IU, MK, JF, SF, IB)

- COMMON VARIABLES

COMMON SHARED IA, SA
COMMON SHARED PI
COMMON SHARED SC, SM, SCR
COMMON SHARED SE, SN, SX, SU, SH, SO, SD, SL, ST, SR, SG, SJ, SW, SDA
COMMON SHARED SP, SXV, SXC, SXM
COMMON SHARED SA8, SI2, SFF
' STATIC ARRAY DEFINITIONS
DIM SFN (12), C(12), CD(12), ER(12), A(12), IR(2, 12), SMN(100), W(12, 100)
, DYNAMIC ARRAY DEFINITIONS
( none)
E ATTRIBUTIONS
SCREEN 12: WINDOW (1, 1)-(640, 480)
' LOGICAL UNIT, EXTENSION AND PATHNAME SETTINGS
$I U=1: I V=2$
SXV = ".CSV": SXC = ".COF": SXM = ".MOD"
$S P=$ "C:\QBASIC\QBFILES\"
' FORMAT DEFINITIONS
SI2 = "\#\#"
SFF = "\#.\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#"
' NUMERICAL CONSTANT DEFINITIONS
$P I=3.141592653589793 \#$

- STRING CONSTANT DEFINITIONS
$\mathrm{SC}=\mathrm{"}: \mathrm{":} \mathrm{SM}=$ ",": $\mathrm{SCR}=\operatorname{CHR} \$(13)+\operatorname{CHR} \$(10)$
' TEXT VARIABLE DEFINITIONS
$S E=$ "ENTER THE ": SN = "NUMBER OF ": SX = "INDEX FOR THE "
ST = "FOURIER TERMS": SH = "HEADER": SO = "ORIGINAL ": SD = "MODEL "
SL = "LOWER ": SU = "UPPER ": SR = "RANK ": SG = "REGRESSION "
SJ = "FILENAME": SW = "SWITCHED": SDA = "DATA"
' ** THE ALGORITHM **

```
        INSCREEN SFO, SHT, SST, K, NM, NX, SFN(), IR()
    MODELCALC IU, K, M, NM, NX, SFO, SFN(), IR(), W(), CD(), ER(), SMN()
    MODELRPT 1, IV, SFO, SFN(), SHT, SST, K, M, IR(), NM, NX, W(), CD(), ER(), SMN()
    NOTE "3C0506"
    NOTE "4D0509"
    NOTE "5E0512"
    NOTE "6F0515"
    END
    SUB DATATRANS (ISW, IU, IV, N, X(), Y())
A SUBROUTINE TO READ OR WRITE THE CO-ORDINATES DATA LIST
    ARGUMENTS:
        ISW THE TRANSPUT SELECTOR SWITCH
            1 READ
                    2 WRITE
                    THE INPUT LOGICAL UNIT NUMBER
                THE OUTPUT LOCICAL UNIT NUMBER
                THE NUMBER OF DATA CO-ORDINATES
                THE ARRAY OF ABSCISSAL CO-ORDINATES
            Y() THE ARRAY OF ORDINAL CO-ORDINATES
    ( EMPLOYED CLICHES ARE COMMON SHARED )
        IF ISW = 1 THEN
            INPUT #IU, N
            FOR I = 1 TO N: INPUT #IU, X(I), Y(I): NEXT I
            ELSE
                    PRINT #IV, N
                    FOR I = 1 TO N: PRINT #IV, X(I); SM; Y(I): NEXT I
            END IF
            END SUB
            SUB DETERMINATION (M, JC, JF, JE, W(), CD())
' A SUBROUTINE TO COMPUTE COEFFICIENTS OF DETERMINATION
BETWEEN COLUMNS OF DATA IN ARRAY W(J,I)
    ARGUMENTS:
            M THE NUMBER OF MINES
            JC THE DETERMINATION COEFFICIENT SERIAL NUMBER
            JF THE COLUMN OF FIDUCIAL REFERENCE DATA
            JE THE COLUMN OF MODEL ESSAY DATA
            W() THE MODEL SERIES MATRIX
            CD() THE ARRAY OF DETERMINATION COEFFICIENTS
            FOR I = 1 TO M: T3 = T3 + W(JF, I): NEXT I: T3 = T3 / M
            FOR I = 1 TO M: U1 = W(JE, I) - T3: U2 = W(JF, I) - T3:T1 = T1 + U1 * U1: T2 = T2 + U2
* U2: NEXT I
            CD(JC) = T1 / T2
            END SUB
            SUB ERRORMETRIC (M, JC, JF, JE, W(), ER())
' A SUBROUTINE TO COMPUTE MODEL ERROR METRICS
, BETWEEN COLUMNS OF DATA IN ARRAY W (J,I)
ARGUMENTS:
            M THE NUMBER OF MINES
            JC THE ERROR METRIC SERIAL NUMBER
            JF THE COLUMN OF FIDUCIAL REFERENCE DATA
            JE THE COLUMN OF MODEL ESSAY DATA
            W() THE MODEL SERIES MATRIX
            ER() THE ARRAY OF MODEL ERROR METRICS
            FOR I = 1 TO M
            D1 =W(JF, I): D2 = (D1 - W(JE, I))
            T2 = T2 + D2 * D2: T1 = T1 + D1 * D1
            NEXT I
            ER(JC) = T2 / T1
            END SUB
    SUB ERRORSET (SER())
' A SUBROUTINE TO DEFINE STANDARD ERROR FLASHES
    SER(1) = "ERROR ONE IS UNDEFINED"
    SER(2) = "ERROR TWO IS UNDEFINED"
    SER(3) = "ERROR THREE IS UNDEFINED"
    SER(4) = "ERROR FOUR IS UNDEFINED"
    SER(5) = "ERROR FIVE IS UNDEFINED"
    SER(6) = "ERROR SIX IS UNDEFINED"
    END SUB
    SUB ERRORSHOW (I, IC, SER())
' A SUBROUTINE TO EXHIBIT THE ERROR MESSAGE SER(I) WITH A WARNING BLEEP
    ARGUMENT :
        I THE ERROR MESSAGE SERIAL NUMBER
```

```
        IC THE COLOR DEFINITION NUMBER
        SER() THE ARRAY OF ERROR DESCRIPTORS
        COLOR IC
        LINEBLANKER 24, 24
        LOCATE 24, 1: PRINT "ERROR " + STR$(I) + ": " + SER(I);
        NOTE "2B0506"
        SLEEP 2
        LINEBLANKER 24, 24
        END SUB
    SUB FOURSERIES (M, M1, M2, N, X(), Y())
A SUBROUTINE TO COMPUTE A SQUARE-WAVE FOURIER SERIES
    ARGUMENTS:
        M THE NUMBER OF RANKED DATA
        M1 THE FIRST RANK FOR WHICH f(x)=1
        M2 THE LAST RANK FOR WHICH f(x)=1
        N THE NUMBER OF FOURIER SERIES TERMS
        X() THE NORMALISED INDEPENDENT VARIABLE
        Y() THE FUNCTION OF X() AT X(I)
    ( PI IS COMMON SHARED )
    Z = (M2 - M1 + 1) / (2 * M)
    FOR I = 1 TO M
        Y(I) = 2 * Z
        FOR J = 1 TO N
            Y(I) = Y(I) + (4 * SIN(J * PI * Z) * COS(J * PI * (Z + (M1 - 1) / M)) * COS(J * PI *
(1 - X(I))) / (J * PI))
            NEXT J
    NEXT I
    END SUB
    SUB GETTER
' A SUBROUTINE TO ACCEPT A KEYSTROKE AS SA AND TO YIELD ITS ASCII CODE AS IA
    ( THE ARGUMENTS SA AND IA ARE COMMON SHARED )
    DO
        SA = INKEY$
    LOOP UNTIL SA <> ""
    IA = ASC (SA)
    END SUB
    SUB HEADERIN (IU, SF, SHT, SST, SXT, SYT)
A SUBROUTINE TO READPOLYNOMIAL FILE HEADER DATA
    ARGUMENTS:
        IU THE INPUT FILE LOGICAL UNIT NUMBER
        SF THE GENERIC FILE NAME
        SHT THE FILE MAIN HEADING
        SST THE FILE SUB HEADING
        SXT THE ABSCISSAL DESCRIPTOR
        SYT THE ORDINAL DESCRIPTOR
    INPUT #IU, SF
    INPUT #IU, SHT
    INPUT #IU, SST
    INPUT #IU, SXT
    INPUT #IU, SYT
    END SUB
    SUB INSCREEN (SFO, SHT, SST, K, NM, NX, SFN(), IR())
' A SUBROUTINE TO ACCEPT FUNDAMENTAL RUN DATA
    ARGUMENTS:
        SFO THE ORIGINAL DATA FILE NAME
        SHT THE OUTPUT MAIN HEADING
        SST THE OUTPUT SUB HEADING
        K THE NUMBER OF CLASSES
        NM THE EXPONENT FOR THE LOWER NUMBER OF FOURIER SERIES TERMS
        NX THE EXPONENT FOR THE UPPER NUMBER OF FOURIER SERIES TERMS
        SFN() THE ARRAY OF POLYNOMIAL COEFFICIENT FILENAMES
        IR() THE ARRAY OF TERMINATION RANK POSITIONS
( SC AND ALL CLICHES ARE COMMON SHARED )
    S1 = "MINEMODL FUNDAMENTAL RUN " + SDA: S2 = STRING$ (29, 45)
    S3 = SE + SJ + "S WITHOUT EXTENSIONS": S4 = SE + SJ + "FOR THE " + SO + SDA + SC
    S5 = SE + "MAIN " + SH + SC: S6 = SE + "SUB " + SH + SC
    S7 = SE + SN + "CLASSES" + SC: S8 = SE + SX + "MINIMUM " + SN + ST + SC
    S9 = SE + SX + "MAXIMUM " + SN + ST + SC: S10 = "CLASS INFORMATION": SI1 = STRING$(17,
45)
    PLACE 4, 0, 0, 0, S1, 2, 26, 1, 2, 0, SF, 0
    PLACE 4, 0, 0, 0, S2, 3, 26, 1, 2, 0, SF, 0
    PLACE 4, 0, 0, 0, S3, 5, 21, 1, 14, 0, SF, 0
    PLACE 4, 0, 0, 0, S4, 7, 9, 1, 2, 0, SF, 0
```

```
    PLACE 4, 0, 0, 0, S5, 8, 9, 1, 2, 0, SF, 0
    PLACE 4, 0, 0, 0, S6, 9, 9, 1, 2, 0, SF, 0
    PLACE 4, 0, 0, 0, S7, 10, 9, 1, 2, 0, SF, 0
    PLACE 4, 0, 0, 0, S8, 11, 9, 1, 2, 0, SF, 0
    PLACE 4, 0, 0, 0, S9, 12, 9, 1, 2, 0, SF, 0
    PLACE 4, 0, 0, 0, S10, 15, 32, 1, 2, 0, SF, 0
    PLACE 4, 0, 0, 0, S11, 16, 32, 1, 2, 0, SF, 0
    PLACE 4, 0, 0, 0, SG, 18, 26, 1, 2, 0, SF, 0
    PLACE 4, 0, 0, 0, SL, 18, 41, 1, 2, 0, SF, 0
    PLACE 4, 0, 0, 0, SL, 18, 41, 1, 2, 0, SF, 0
    PLACE 4, 0, 0, 0, SU, 18, 51, 1, 2, 0, SF, 0
    PLACE 4, 0, 0, 0, SJ, 19, 27, 1, 2, 0, SF' 0
    PLACE 4, 0, 0, 0, SR, 19, 41, 1, 2, 0, SF, 0
    PLACE 4, 0, 0, 0, SR, 19, 51, 1, 2, 0, SF, 0
    PICK 4, 0, 0, 0, SFO, 7, 53, 3, SA8
    PICK 4, 0, 0, 0, SHT, 8, 32, 3, SA40
    PICK 4, 0, 0, 0, SST, 9, 32, 3, SA40
    PICK 1, K, 0, 0, S, 10, 38, 3, SI2
    PICK 1, NM, 0, 0, S, 11, 66, 3, SI2
    PICK 1, NX, 0, 0, S, 12, 66, 3, SI2
    FOR I = 1 TO K
        IW = I + 20
        PICK 4, 0, 0, 0, SFN(I), IW, 27, 3, SA8
    PICK 1, IR(1, I), 0, 0, S, IW, 42, 3, SI2
    PICK 1, IR(2, I), 0, 0, S, IW, 52, 3, SI2
NEXT I
END SUB
    SUB LINEBLANKER (I, J)
' A SUBROUTINE TO BLANK SCREEN LINES I TO J INCLUSIVE
    ARGUMENTS:
    I THE START LINE
    J THE FINISH LINE
    FOR K = I TO J: LOCATE K, 1: PRINT SPACE$(80); : NEXT K
    END SUB
    SUB META (M, SM)
' A SUBROUTINE TO NOTIFY THE CURRENT STAGE OF ELABORATION
    ARGUMENTS:
        M THE ARBITRARY STAGE NUMBER
                        ( NEGATIVE FOR OMISSION )
        SM THE STAGE DESCRIPTOR
        IF M > -1 THEN
            LOCATE 1, 1: COLOR 2: PRINT "META"
            LOCATE 1, 6: COLOR 14: PRINT M
            LOCATE 1, 10: PRINT SPACE$ (80): LOCATE 1, 10: COLOR 12: PRINT SM
        END IF
        END SUB
    SUB MODELCALC (IU, K, M, NM, NX, SFO, SFN(), IR(), W(), CD(), ER(), SMN())
' A SUBROUTINE TO CONSTRUCT THE FOURIER-POLYNOMIAL MODEL FITMENTS
' WITH THEIR RESPECTIVE MODEL ERROR METRICS
    ARGUMENTS:
        IU THE INPUT LOGICAL UNIT NUMBER
        K THE NUMBER OF CLASSES
        M THE NUMBER OF MINES
        NM THE EXPONENT FOR THE LOWER NUMBER OF FOURIER SERIES TERMS
        NX THE EXPONENT FOR THE UPPER NUMBER OF FOURIER SERIES TERMS
        SFO THE ORIGINAL DATA FILE NAME
        SFN() THE ARRAY OF POLYNOMIAL COEFFICIENT FILENAMES
        IR() THE ARRAY OF TERMINATING RANK POSITIONS
        W() THE MODEL SERIES MATRIX
        CD() THE ARRAY OF DETERMINATION COEFFICIENTS
        ER() THE ARRAY OF MODEL ERROR METRICS
        SMN() THE ARRAY OF MINE NAMES
DIMENSION THE LOCAL ARRAYS
    DIM ID(12), CC(12, 12), X(100), Y(100)
' LOAD THE ORIGINAL RANK VERSUS LOG PRODUCTION DATA
    OPEN "I", IU, SP + SFO + SXV
    HEADERIN IU, SF, SHT, SST, SXT, SYT
    INPUT #IU, M
    FOR I = 1 TO M: INPUT #IU, SMN(I), X(I), Y(I): NEXT I
    CLOSE IU
    FOR I = 1 TO M: W(1, I) = Y(I): NEXT I
    ' LOAD THE POLYNOMIAL REGRESSION DEGREES AND COEFFICIENTS
    FOR I = 1 TO K
        OPEN "I",'IU, SP + SFN(I) + SXC
        HEADERIN IU, SF, SHT, SST, SXT, SYT
        DATATRANS 1, IU, IV, ID(I), X(), Y()
        CLOSE IU
```

FOR $J=1$ TO ID(I): CC(J, I) = Y(J): NEXT J
NEXT I
' DEFINE THE ARRAY OF NORMALISED RANKS
FOR I = 1 TOM: X(I) = (M - I) / (M - 1): NEXT I
' ESTABLISH THE POLYNOMIAL SEGMENTS BASE MODEL
FOR $I=1$ TO K
$\operatorname{FOR} \mathrm{J}=\operatorname{IR}(1, \mathrm{I}) \quad \operatorname{TO} \operatorname{IR}(2, \mathrm{I})$
$W(2, J)=0!: F O R J J=1$ TO ID (I) : W(2, J) $=W(2, J)+C C(J J, I) * X(J) \wedge(J J-$
1): NEXT JJ

NEXT J
NEXT I
' COMPUTE THE DETERMINATION COEFFICIENT AND THE MODEL ERROR METRIC
' ON POLYNOMIALS ALONE
DETERMINATION M, 0, 1, 2 , W(), CD()
ERRORMETRIC M, $0,1,2, W(), \operatorname{ER()}$
' COMPUTE THE FOURIER-POLYNOMIAL MODEL FITMENTS

- AND THEIR ERROR METRICS

II $=2$
FOR $I=1$ TO $M: X(I)=(M-I) /(M-1): N E X T I$
FOR I $=$ NM TO NX
$I I=I I+1: N U=2 \wedge I$
FOR J = 1 TO K
FOURSERIES M, IR(1, J), IR(2, J), NU, X(), Y()
FOR JJ $=1 \mathrm{TO} \mathrm{M}$
$Z=0!: F O R K K=1$ TO ID(J): $Z=Z+C C(K K, J) * X(J J) \wedge(K K-1):$ NEXT KK
$W(I I, J J)=W(I I, J J)+Z$ * Y(JJ)
NEXT JJ
NEXT J
ERRORMETRIC M, II - 2, 1, II, W(), ER()
NEXT I
END SUB
SUB MODELRPT (ISW, IV, SFO, SFN(), SHT, SST, K, M, IR(), NM, NX, W(), CD(), ER(), SMN () )
' A SUBROUTINE TO OUTPUT THE FOURIER POLYNOMIAL FITMENTS
ARGUMENTS:
ISW THE OUTPUT DESTINATION SWITCH
0 OUTPUT TO THE SCREEN
1 OUTPUT TO THE NOMINATED FILE
IV THE OUTPUT LOGICAL UNIT NUMBER
SFO THE ORIGINAL DATA FILE NAME
SFN() THE ARRAY OF POLYNOMIAL COEFFICIENT FILENAMES
SHT THE OUTPUT MAIN HEADING
SST THE OUTPUT SUB HEADING
K THE NUMBER OF CLASSES
M THE NUMBER OF MINES
IR() THE ARRAY OF TERMINATING RANK POSITIONS
NM THE EXPONENT FOR THE LOWER NUMBER OF FOURIER SERIES TERMS
NX THE EXPONENT FOR THE UPPER NUMBER OF FOURIER SERIES TERMS
W() THE MODEL SERIES MATRIX
CD() THE ARRAY OF DETERMINATION COEFFICIENTS
ER() THE ARRAY OF MODEL ERROR METRICS
SMN () THE ARRAY OF MINE NAMES
( SP, SC, SM, SCR AND ALL CLICHES ARE COMMON SHARED )

```
IF ISW = 0 THEN SOU = "CON" ELSE SOU = SP + SFO + SXM
OPEN "O", IV, SOU
SS = STRING$(41, 42)
PRINT #IV, SS
PRINT #IV, "A WORKSHEET TO PRESENT FOURIER-POLYNOMIAL"
PRINT #IV, " MODEL FITMENTS TO LEAD MINE"
PRINT #IV, " LOG PRODUCTION VERSUS RANK " + SDA
PRINT #IV, SS + SCR + SCR
PRINT #IV, "JAMES R WARREN" + SCR + SFO + SXM + SCR + DATE$ + SCR + SCR
PRINT #IV, SHT + SCR + SST + SCR + SCR
PRINT #IV, SN + "CLASSES" + SM; K
PRINT #IV, SCR + SO + SDA + SM + SFO + SXV
PRINT #IV, SN + "MINES" + SM; M
PRINT #IV, SCR + SCR + "POLYNOMIAL SEGMENTS" + SCR + STRING$(19, 45) + SCR + SCR
PRINT #IV, SG + SM + SL + SM + SU
PRINT #IV, SJ + SM + SR + SM + SR
PRINT #IV,
FOR I = 1 TO K
    PRINT #IV, SFN(I) + SXC + SM; IR(1, I); SM; IR(2, I)
    NEXT I
PRINT #IV, SCR + SCR + "THE TABLE OF MODEL APPROXIMATIONS" + SCR + STRING$(33, 45) +
    PRINT #IV, SM + "MINE" + SM + SO + SM + "LOGICALLY" + SM + "FOURIER"
    PRINT #IV, SM + "NAME" + SM + SDA + SM + SW + SM + SW
    PRINT #IV, SM + SM + SM + SD
    PRINT #IV, "NUMBER OF TERMS = " + SM + SM + SM;
    FOR I = NM TO NX
```

$S C R+S C R$

PRINT \#IV, SM; $2 \wedge \mathrm{I}$;
NEXT I
PRINT \#IV, SCR
$J J=N X-N M+3$
FOR I = 1 TO M
PRINT \#IV, $\mathrm{SM}+\mathrm{SMN}(\mathrm{I})$;
FOR J $=1 \mathrm{TO} \mathrm{JJ}$
PRINT \#IV, SM;
PRINT \#IV, USING SFF; W(J, I)
NEXT J
PRINT \#IV,
NEXT I
PRINT \#IV, SCR + SCR
PRINT \#IV, "DETERMINATION COEF." + SM + SM;
PRINT \#IV, USING SFF; 1;
PRINT \#IV, SM;
PRINT \#IV, USING SFF; CD(0)
PRINT \#IV, "MODEL ERROR METRICS" + SM + SM;
PRINT \#IV, USING SFF; 0;
FOR $I=0 \mathrm{TO} \mathrm{NX}-\mathrm{NM}+1$
PRINT \#IV, SM;
PRINT \#IV, USING SFF; ER(I);
NEXT I
PRINT \#IV,
CLOSE IV
END SUB
SUB NOSEG (SK)
A SUBROUTINE TO FLASH THE CAPTION "SEGMENT SK UNAVAILABLE"
' BEFORE A WARNING BUZZ
ARGUMENT:
SK THE MISSING SEGMENT OR PROCESS DESCRIPTOR
LINEBLANKER 24, 24
COLOR 6
LOCATE 24, 1: PRINT "SEGMENT ";
COLOR 14: PRINT SK;
COLOR 6: PRINT " UNAVAILABLE";
NOTE "2B0506"
SLEEP 2
LINEBLANKER 24, 24
END SUB
SUB NOTE (S)
SUBROUTINE TO SOUND A NOTE UPON THE COMPUTER SPEAKER
ARGUMENT:
S THE NOTE SPECIFIER STRING "IN\$NL" e.g. "2B0506"
I THE OCTAVE NUMBER ( $0-6$ )
N\$ THE NOTE LETTER ( ABCDEFG )
N THE NOTE NUMBER ( $0-84$ )
L THE LENGTH OF THE NOTE ( 1-64)
$I=\operatorname{VAL}(\operatorname{MID} \$(S, 1,1)): N \$=\operatorname{MID}(S, 2,1)$
$\mathrm{N}=\operatorname{VAL}(\operatorname{MID}(\mathrm{S}, \mathrm{3}, 2)): \mathrm{L}=\operatorname{VAL}(\operatorname{MID}(\mathrm{S}, 5,2))$
PLAY "O" + STRS (I) + "N" + STRS (N) + "L" + STRS (L) + "X" + VARPTRS (N\$)
END SUB
SUB PICK (KT, I, L, R, S, IY, IX, MK, SF)
SUBROUTINE TO OBTAIN A VARIABLE OF TYPE KT AT SCREEN POSITION IY, IX
ARGUMENTS:
KT THE DATUM TYPE CHOICE
1 SHORT INTEGER

3 DOUBLE PRECISION REAL
$\begin{array}{ll}3 & \text { DOUBLE } \\ 4 & \text { STRING }\end{array}$
THE SHORT INTEGER
THE LONG INTEGER TO BE OBTAINED ( OPTION )
THE REAL TO BE OBTAINED ( OPTION )
THE STRING TO BE OBTAINED ( OPTION )
THE STARTING SCREEN ROW
THE STARTING SCREEN COLUMN
THE PRINTING COLOR
THE PROPER ( OR DEFAULT ) PRINTING FORMAT
( "SAnn" TRUNCATES A STRING TO nn CHARACTERS )
SDEL $=$ CHR\$ (0) $+\operatorname{CHR} \$(83)$
COLOR MK: LOCATE IY, IX: IO = I: LO = L: RO = R: SOL = S: L1 = LEN (SF)
SELECT CASE KT
CASE 1
IF LEN(STR\$(I)) > L1 THEN L1 = LEN (STR\$ (I))
CASE 2
IF LEN (STR\$ (L)) > L1 THEN L1 = LEN (STR\$ (L))

```
    CASE 3
        IF LEN(STR$(R)) > L1 THEN L1 = LEN(STR$(R))
    CASE 4
        L1 = LEN(S)
    END SELECT
    PRINT SPACE$(L1)
    DO
    LOCATE IY, IX: PRINT "."
    FOR II = 1 TO 10: NEXT II
    LOCATE IY, IX: PRINT " "
    SA = INKEY$
    LOOP UNTIL SA <> ""
    IA = ASC (SA)
    IF IA <> }13\mathrm{ THEN
    SCON = "": IT = IX: LOCATE IY, IX
        DO
            IF SA = SDEL THEN
                IT = IT - 1
                        LOCATE IY, IT: PRINT SPACE$(1)
                        LOCATE IY, IT: SCON = LEFT$(SCON, LEN(SCON) - 1)
                ELSE
                        SELECT CASE KT
                            CASE 1 TO 3
                            IF IA > 47 AND IA < 58 OR IA = 46 OR IA = 45 THEN
                                    SCON = SCON + SA
                                    PRINT SA;
                                    IT = IT + 1
                                    ELSE
                                    IF IA <> 13 THEN NOTE "2B0506"
                                    END IF
                                    CASE 4
                                    IF IA > 31 AND IA < 127 THEN
                                    SCON = SCON + SA
                                    PRINT SA;
                                    IT = IT + 1
                            ELSE
                                    IF IA <> 13 THEN NOTE "2B0506"
                                    END IF
                            END SELECT
            END IF
            GETTER
        LOOP UNTIL IA = 13
        SELECT CASE KT
            CASE 1
                I INT (VAL (SCON) + .5)
            CASE 2
                L = INT (VAL (SCON) + .5)
                    CASE 3
                R = VAL (SCON)
            END SELECT
    ELSE
        I = IO: L = LO: R = RO
    END IF
    L1 = LEN(SF): IF LEN(SCON) > L1 THEN L1 = LEN(SCON)
    LOCATE IY, IX: PRINT SPACE$(L1): LOCATE IY, IX
    SELECT CASE KT
    CASE 1
        PRINT USING SF; I
    CASE 2
        PRINT USING SF; L
    CASE 3
        PRINT USING SF; R
    CASE 4
        LOCATE IY, IX: PRINT SPACE$(LEN(SCON)): LOCATE IY, IX
        IF SOL <> "" AND SCON = "" THEN SCON = SOL
        IF LEFT$(SF, 2) = "SA" THEN
            S = LEFT$(SCON, VAL (MID$(SF, 3)))
            ELSE
                S = SCON
        END IF
        PRINT S
    END SELECT
    END SUB
    SUB PLACE (KT, I, L, R, S, IY, IX, IU, MK, JF, SF, IB)
A SUBROUTINE TO PLACE A VARIABLE OF TYPE KT AT SCREEN POSITION IY, IX
OR ALTERNATIVELY PLACE THE VARIABLE WITHIN A PADDED REPORT FILE
    ARGUMENTS:
        KT THE DATUM TYPE CHOICE
                        1 SHORT INTEGER
                        LONG INTEGER
    DOUBLE PRECISION REAL
```

```
            STRING
        THE SHORT INTEGER TO BE PRINTED ( OPTION )
        THE LONG INTEGER TO BE PRINTED ( OPTION )
        THE REAL TO BE PRINTED ( OPTION )
        THE STRING TO BE PRINTED (OPTION)
        THE STARTING SCREEN ROW
        THE STARTING SCREEN COLUMN
        THE LOGICAL UNIT NUMBER
            PRINT TO THE SCREEN
            2 PRINT TO A REPORT FILE
        THE NOMINAL PRINTING COLOR
        THE LINE FEED SUPPRESSOR SWITCH
            FOLLOW WITH A LINE FEED
            1 DO NOT FOLLOW WITH A LINE FEED
    THE REQUIRED PRINTING FORMAT
    THE NUMBER OF FORWARD PADDING SPACES
IF SF = "" THEN SF = "#########"
COLOR MK
SELECT CASE IU
    CASE 1
        LOCATE IY, IX
            SELECT CASE KT
                    CASE 1
                    PRINT USING SF; I
                    CASE 2
                    PRINT USING SF; L
                    CASE }
                    PRINT USING SF; R
                    CASE 4
                    PRINT S
            END SELECT
    CASE 2
        IF IB > 0 THEN PRINT #IU, SPACE$(IB);
        IF JF = 1 THEN SCC = ";" ELSE SCC = CHR$(13) + CHR$(10)
            SELECT CASE KT
            CASE 1
                PRINT #IU, USING SF; I; SCC
            CASE 2
                PRINT #IU, USING SF; L; SCC
            CASE 3
                PRINT #IU, USING SF; R; SCC
            CASE 4
                PRINT #IU, S
                END SELECT
IF JF = 0 THEN PRINT #LU,
END SELECT
END SUB
```


[^0]:    -Original Data - Logically Switched -Fourier 64 -Fourier 128 -Fourier 256 -Fourier 512 -Fourier 1024

