A Snail Survives an Epic Journey in My Hubcap

by
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Abstract

Some Helix Aspersa snails attached themselves to various semi-concealed parts of a parked automobile. Before they could safely be removed they survived various hostile and unnatural environments including an automatic car wash. Other snails cemented themselves to hub-caps and endured high-speed thousand-kilometer journeys on British motorways, apparently surviving the return to their home habitat. A mathematical analysis of the dynamics endured is elaborated in this celebration of Europe's sturdy Garden Snail.

Before my garage is an asphalt driveway shaded by my neighbor's laburnum trees. When the ground is damp, in Spring and Autumn especially, flocks of European Garden Snails (*Helix Aspersa*) scale the small brick wall beside the trees and promenade across the wet pavement in gentle quest of espousal. Frequently they climb the garage doors and walls to nestle in the cornice corners three meters from the ground or even essay little journeys into the dark and humid recesses of the building.

Snails have very strong opinions about where they should be. A tiny individual, five millimeters in circumference, sat two meters above the drive on brick next to the entry door. Unfortunately, I dislodged him as I closed the door and he plummeted to the asphalt below: Within twenty-four hours he had resumed his exact position and orientation and apparently retracted to sleep.

When my Father came to visit at Whitsun 2000 we parked his trailer in the garage and our cars on the drive beneath the flowering laburnums, where the vehicles became encrusted with gum and blossom. After Father left we took our car to the wash. Imagine our surprise when a few minutes later we lifted the hood and found twelve large snails sleeping in the spandrels of the gutter apparently unmoved by hot soapy water and hot air. My Wife Jana rescued them and returned them to their homeland beneath the priestess's laburnums.

The subsequent July, Jana and I drove the 415 miles (668 km) from Bloxwich to my Father's croft at Netherley in Scotland. On Thursday 20 July we went to Aberdeen to have the front wheels balanced. My Wife checked all four wheels and saw nothing unusual. But the subsequent Saturday morning we noticed that a large, elderly snail (33 mm long by 15 broad and 18 high) had neatly ensconced himself in one of the twenty-nine cooling louvers of my nearside rear hubcap.

The hubcap appears to me to be made of polypropylene coated with a film to simulate brushed aluminum. This film carries numerous scratches where I have scraped the curb, but the small square tunnel where the snail lodged is smooth and greasy.

We gingerly tried to extract him from the vent but we had to be careful not to break his delicate shell with our fingers, and it seemed to us that whenever we made contact he defiantly made firm his implacable adhesion to the wheel. When snails are walking it is possible gently to lift them clear of a wet surface, whereupon they withdraw to their shells. But when a snail is dormant he seals himself behind a hard operculum and cements the lip of his shell to his chosen bed. The broad smear of mucus and feces across the retaining rim of the tire arbor confirmed our impression that the shell had a living inhabitant but reluctantly we had to allow him to stay lest we injured him in removal.

Photograph 1
Snails on the a Volvo Hub-Cap at Bloxwich
am 28 July 2000
The animal in the louver traveled to Scotland and back

Accordingly, he sat in his louver during subsequent drives around Deeside and on our homeward journey to Bloxwich via Inverkip, Renfrewshire, where we stayed on the night of Wednesday 26 July after visiting friends in Dunoon. Therefore, the snail traveled 600 miles (966 km) on my hubcap whilst the car cruised the motorway parts of the return at 80 mph (129 km/hr) touching 94 mph (151 km/hr) through parts of North-West England.

When we reached Bloxwich I returned the car to the garage. As I removed the hubcap a small pebble or something fell from the dish onto the concrete floor. When I picked it up I was astonished to hold a fair-shelled young snail (17.5 mm long by 14 broad and 9 high) who was definitely in his shell behind a wet, glistening operculum. I put the hubcap near the door and dowsed it with water, placing the little one by the pool. The older snail clung fast to his louver.

Jana came and we gingerly detached him only to find an empty shell that had partly cretacified in the manner of old shells abandoned by these animals. I had very much wanted the old snail to survive his horrendous travels and Jana said that he had vacated his shell and crawled away without it, but I thought she said this only to console me.

She then moved the little one to the dark corner of the garage and left him on the dry concrete floor explaining that he would find his own way overnight.

I returned the next morning to find the shell had moved about twenty centimeters to a narrow crevice between the floor and the front wall inside the garage. The little shell was empty.

It seems incredible that anything could survive periodically very hot winds howling through the louver from the brakes at up to 21.7 ms^{-1} (48.5 mph) and accelerations alternating between zero and 7.2 g at 40.6 Hz, sustained through some 548,861 revolutions of the car wheel.

Yet such is what the following analysis reveals, and though performed from the perspective of the large snail in the cooling louver, it would hold more or less for the little one who dwelt somewhere on the hub.

Data

I drive a 1991 Volvo 460 GLE sports sedan fitted with a Goodyear Eagle Ventura 185/60R14 82H, inflated to 27 psi, on the rear nearside. (I.e. The *left* side of the car: Remember the British drive on the left).

The Wheel Diameter with this tire fitted is 560 mm and the Radial Displacement of The Large Snail's Centroid (within the hubcap cooling louver) was 110 mm from the tire periphery. Hence his Radial Displacement from the Hub Center was 170 mm. The Wheel Radius is of course 280 mm.

As aforementioned, the two snails found had the following shell dimensions in millimeters:-

Snail	Length	Width	Spire Height
A	33	25	18
B	17.5	14	9

There are 1609.344 meters in a mile and The Acceleration due to Gravity is assumed to be 9.8062 ms^{-2}.

The analysis assumed a nominal car speed of 80 mph (35.7632 ms^{-1} or 128.74752 km/hr) and a journey of 600 miles (965606.4 meters).

Forces along the Trochoid

An object placed eccentrically along a radius describes a trigonometric curve called a trochoid as the circle to which it is affixed (i.e. the wheel) rolls along the horizon without slippage.

Such an object experiences a local centrifugal force different to that experienced upon a non-traveling rotor (for instance a lathe arbor).

This Centrifugal Acceleration, w, is however definable as:-

$$w = \frac{v^2}{r} \qquad \textbf{Eqn.1}$$

where in context v is the Local Particle (Object) Velocity relative to static ground and r is the Local Radius of Trochoid Curvature.

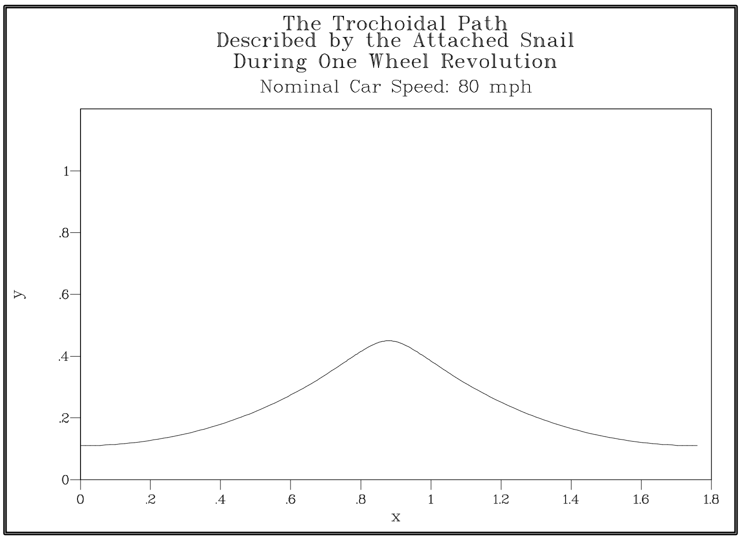

Figure 1

The Geometry of the Trochoid

The geometry of the trochoid, illustrated for any one cycle by Figure One, is given by the parametric equations:-

$$x = R\theta + a\,\mathrm{Sin}\,\theta \qquad \textbf{Eqn.2}$$

$$y = R - a\,\mathrm{Cos}\,\theta \qquad \textbf{Eqn.3}$$

Now v and r are definable in terms of ω, The Angular Velocity of the Wheel, together with first and second differentials of Equations Two and Three.

The Angular Velocity is given by:-

$$\omega = \frac{V}{2\pi R} \qquad \textbf{Eqn.4}$$

where V is the Vehicle Speed and R is the Wheel Radius. For V=80 mph, ω=20.32817878851454 Hz. This sixteen-figure result can be used for checking purposes.

The first and second differentials of the parametric equations of the trochoid are:-

$$\frac{dx}{d\theta} = R + a\,\mathrm{Cos}\,\theta \qquad \textbf{Eqn.2a}$$

$$\frac{d^2 x}{d\,\theta^2} = -\,a\,\mathrm{Sin}\,\theta \qquad \textbf{Eqn.2b}$$

and:-

$$\frac{dy}{d\theta} = a\,\mathrm{Sin}\,\theta \qquad \textbf{Eqn.3a}$$

$$\frac{d^2 y}{d\,y^2} = a\,\mathrm{Cos}\,\theta \qquad \textbf{Eqn.3b}$$

Particle Velocity, v, is given by:-

$$v = \omega\sqrt{\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2} \qquad \textbf{Eqn.5}$$

whence:-

$$v^2 = \omega^2 \left[\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2\right] \qquad \textbf{Eqn.6}$$

whilst Curvature Radius, r, is given by:-

$$r = \frac{\left[\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2\right]^{\frac{3}{2}}}{\left|\frac{dx}{d\theta}.\frac{d^2y}{d\theta^2} - \frac{d^2x}{d\theta^2}.\frac{dy}{d\theta}\right|} \qquad \textbf{Eqn.7}$$

Accordingly, Local Acceleration, w, is:-

$$w = \frac{v^2}{r} = \frac{\omega^2\left[\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2\right]}{\frac{\sqrt{\left[\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2\right]^3}}{\left|\frac{dx}{d\theta}.\frac{d^2y}{d\theta^2} - \frac{d^2x}{d\theta^2}.\frac{dy}{d\theta}\right|}} \qquad \textbf{Eqn.8}$$

which simplifies to:-

$$w = \frac{\omega^2}{\sqrt{\left[\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2\right]}}.\left|\frac{dx}{d\theta}.\frac{d^2y}{d\theta^2} - \frac{d^2x}{d\theta^2}.\frac{dy}{d\theta}\right| \qquad \textbf{Eqn.9}$$

Since ω is directly proportional to V it is clear that the severity of the centrifugal accelerations varies as the square of the car speed.

In order to establish the actual Force, F, impinging upon the snail we would have to multiply w by the animal's mass, m, according to:-

$$F = mw \qquad \textbf{Eqn.10}$$

I have not attempted to estimate the mass of the living mollusks in their shells. Their densities would be about 1030 kgm^{-3}. Crude estimates of shell volumes may be assessed using:-

$$C = \frac{\pi}{6}\, x\, BHL \qquad \textbf{Eqn.11}$$

where C is the Shell Volume and B, H and L are respectively Breadth, Spire Height and Length. This treats the snail as a sphere whose radius is the geometric mean of half its orthogonal dimensions. Thus the larger, louver-dwelling creature, Snail A, has a volume of some 7.8 milliliters and the smaller Snail B 1.2 milliliters.

Particle Velocity, v, varies with Wheel Angle of Rotation, θ. This speed along the trochoid trajectory is surprisingly low reaching a maximum of about 9 ms^{-1} at θ=0 and 2π and a minimum of about 2.5 at θ=π. The mean is only about 6ms^{-1}. Figure Two, a quasi-sinusoidal geometry, expresses v^2, whose extrema are of course identically located on θ.

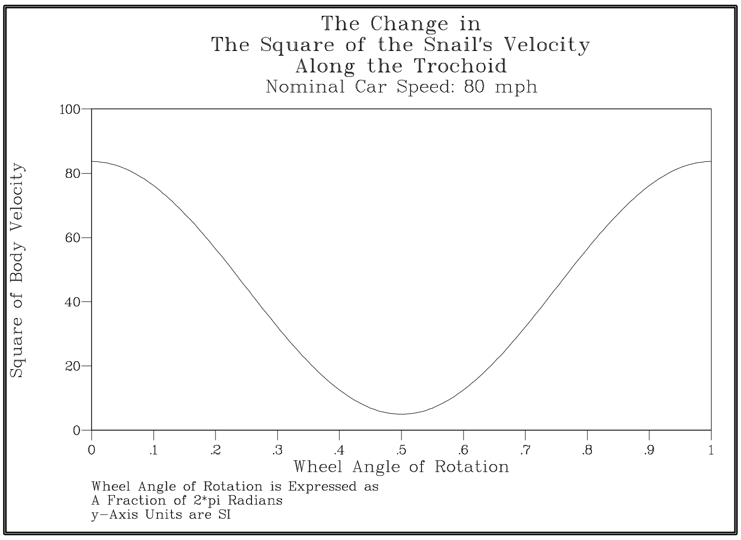

Figure 2
The Variation of Velocity Squared

Figure Three is wholly different, expressing the value of Curvature r. The twin peaks, numerically at r≈40 meters, are in fact at infinite radii in the inflections of the trochoid, though typical radii of curvature are more nearly 1.17 meters, declining to 0.0711765 meters at θ=π radians.

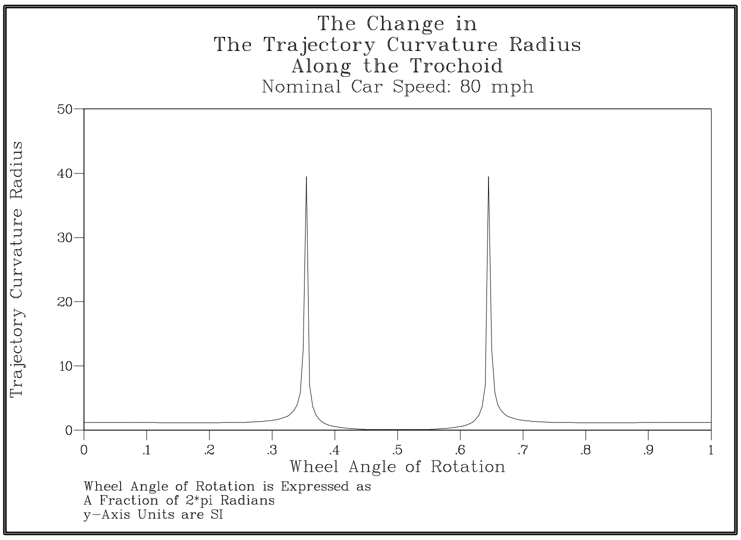

Figure 3
The Variation of Curvature

Figure Four shows the combined effect of v^2 and r on the Particle Acceleration, w. At two points symmetric about θ=π, w vanishes and the snail becomes "weightless" in terms of trochoidal dynamics. (He remains of course subject to the Earth's gravitational acceleration, whose component, by the way, is neglected in this study). At θ=π itself, and also at θ=0 or 2π, acceleration attains its maximum value of precisely 70.24992498582816 ms^{-2} or 7.163827475049266 g. Study of Figure Four shows that in each wheel revolution there are two distinct accelerative cycles, dissimilar both in period and in geometry. It is possible for one or both of these to drive resonant destruction of a suitably-sized and suitably-located object.

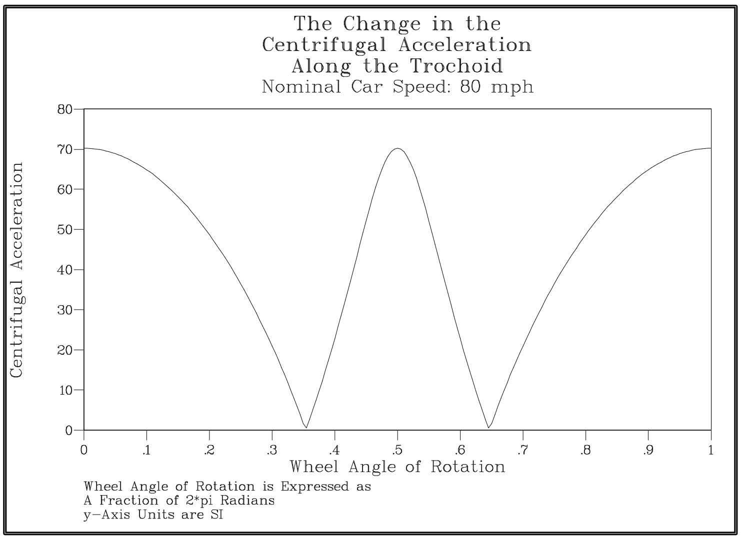

Figure 4
The Variation of Acceleration

<u>The Locations of Absent Acceleration</u>

To define the values of θ at which w=0 we should first explicate Equation Nine by the substitution of the differentials' terms:-

$$w = \frac{\omega^2}{\sqrt{(R + a\,\mathrm{Cos}\,\theta)^2 + (\mathrm{aSin}\,\theta)^2}} \left[(\mathrm{R} + \mathrm{aCos}\,\theta).(\mathrm{aCos}\,\theta) - (-\,\mathrm{aSin}\,\theta).(\mathrm{aSin}\,\theta)\right] \quad \textbf{Eqn.12}$$

which may be simplified to:-

$$w = \frac{T_1}{T_2}.T_3 = \frac{\omega^2}{\sqrt{R^2 + 2R\,\mathrm{Cos}\,\theta + \mathrm{a}^2}}.\left|\ \mathrm{RaCos}\,\theta + \mathrm{a}^2\ \right| \quad \textbf{Eqn.13}$$

where:-

$$T_1 = \omega^2 \quad \textbf{Eqn.13a}$$

$$T_2 = \sqrt{R^2 + 2RCos\theta + a^2} \qquad \textbf{Eqn.13b}$$

$$T_3 = \left| RaCos\theta + a^2 \right| \qquad \textbf{Eqn.13c}$$

Now at w=0, θ=ψ and we may re-cast Equation Thirteen as:-

$$0 = \frac{\omega^2}{\sqrt{R^2 + 2R\,\mathrm{Cos}\,\psi + \mathrm{a}^2}} . \left| \ \mathrm{RaCos}\,\psi + \mathrm{a}^2 \ \right| \qquad \textbf{Eqn.14}$$

Further, T_1 cannot be zero since the wheel is turning. T_2 cannot be zero because R and a are finite and the equality must be zero. Therefore, T_3 must be zero and such can only eventuate if:-

$$a^2 = -Ra\,\mathrm{Cos}\,\psi \qquad \textbf{Eqn.15}$$

Hence, when w=0:-

$$\psi = \mathrm{ArcCos}\left[\frac{\mathrm{a}^2}{-\mathrm{Ra}}\right] = \mathrm{ArcCos}\left[\frac{\mathrm{a}}{-\mathrm{R}}\right] \qquad \textbf{Eqn.16}$$

Given the data values of R and a, ψ and 2π-ψ are respectively 2.223256224112974 or 4.059929083066613 radians.

<u>The Equation for Maximum Acceleration</u>

Maximum Acceleration occurs at 0 (or 2π) radians; and also at π radians.
Note that when θ=0, Cosθ=1. When θ=π, Cosθ=-1.
In the former case:-

$$\begin{aligned} w_{\max} &= \frac{\omega^2}{\sqrt{R^2 + 2Ra + a^2}} . \left| \ Ra + a^2 \ \right| \\ &= \frac{\omega^2}{\sqrt{(R+a)(R+a)}} .a.(R+a) \end{aligned} \qquad \textbf{Eqn.17}$$

or:-

$$\begin{aligned} w^2_{\max} &= \frac{\omega^4}{(R+a)(R+a)} . a^2 . (R+a)^2 \\ &= \omega^4 a^2 \end{aligned} \qquad \textbf{Eqn.18}$$

Hence:-

$$w_{\max} = \sqrt{\omega^4 a^2} = a\,\omega^2 \qquad \textbf{Eqn.19}$$

Now for the case of θ=π:-

$$w_{\max} = \frac{\omega^2}{\sqrt{R^2 - 2Ra + a^2}} . \left| \; - Ra + a^2 \; \right|$$
$$= \frac{\omega^2}{\sqrt{(R-a)(R-a)}} . a . (R - a) \qquad \textbf{Eqn.20}$$

or:-

$$w_{\max}^2 = \frac{\omega^4}{(R-a)(R-a)} . a^2 . (R-a)^2$$
$$= \omega^4 a^2 \qquad \textbf{Eqn.21}$$

Hence:-

$$w_{\max} = \sqrt{\omega^4 a^2} = a\,\omega^2 \qquad \textbf{Eqn.22}$$

Equations Nineteen and Twenty-Two yield the identical value of 70.24992498582813 ms^{-2}; also identical to the value calculated using Equation Thirteen.

$w = a\omega^2$ is identical to the equation $w = r\omega^2$ for the centrifugal acceleration at radius r on a non-traveling rotor.

The Snail

This essay is a celebration of the snail and his tenacity of life. Languid but never indolent his three brains unfailingly afford him a decisive election, whatever men may think of the wisdom of his choices.

He can endure and survive hellish forces and privations unimaginable yet easily-calculable by men.

A reputed crop pest, I have never seen the snail devour leaves or flowers, or found evidence thereof.

Gregarious but never dependent, social but never contentious, the European Garden Snail offends no man or beast, grazing algae and a little calcareous pointing he and his family adorn rather than corrode the appointments of our houses.

Notation

a	The Radial Displacement of the Particle Mass Centroid
B	The Snail Breadth
C	The Snail Shell Volume
F	The Force Acting upon Any Particle
g	The Acceleration Due to Gravity
θ	The Wheel Angle of Rotation
H	The Snail Spire Height
L	The Snail Length
m	The Particle Mass
π	The Ludolphine Constant
r	The Local Radius of Trochoid Curvature
R	The Wheel Radius
T_n	The nth. Defined Term
v	The Local Particle Velocity along the Trochoid
V	The Vehicle Speed
w	The Local Centrifugal Acceleration
w_{max}	The Maximum Centrifugal Acceleration
x	The Abscissal (i.e. road-wise) Displacement
y	The Ordinal (i.e. perpendicular to road) Displacement
ω	The Angular Velocity of the Wheel

Software

Equation differentiations and simplifications were assisted by MathCad 6.0 Student Edition.

Plots were drawn using SuperCalc 5.

Reference

"The Penguin Dictionary of Mathematics"
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