Some Approaches to the Determination of The Mean Transect of A Regular Polygon

by
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A Transect of a Regular Polygon is a specific straight line between fixed points on two different sides of the figure.

The Mean Transect is the average length (defined as an arithmetic mean) of the assemblage of random transects of the polygon concerned. It is convenient to standardise this length as a ratio to a side of unit length.

The Mean Transect is a characteristic constant for any particular n-sided polygon. Nevertheless, an analytic value is available for only the n=4 polygon (the square). In all other cases approximations must be established by numerical or statistical means.

PART I

THE ANALYTIC MEAN TRANSECT OF A SQUARE

In general, if a mean may be identified with the expectation of variable x then:-

$$E(x) = \int_{-\infty}^{\infty} x.f(x).dx \qquad \textbf{Eqn.1}$$

Allow that the Mean Transect Length, h_μ, is the mean straight line distance between arbitrary points on two different sides of a square with sides of length unity.

Because the square is a regular polygon symmetry allows that we may project such TRANSECTS from the base only to reach:-

(a) The Left-Hand Side
(b) The Top Side
(c) The Right-Hand Side

and the generated probability-weighted averages will represent starts and finishes on any two different sides.

Such TRANSECTS may analytically be conceived as radii projected for an infinite series of points, b, along the basal side, which should be doubly-integrated: For the destination of projection; and then for the origin of projection, b. Both these integrals are definite on the interval (0,1) because each side is of unit length. In the first instance, we require to compute individual transects, h, as their squares h^2 because it is more convenient to employ Pythagoras Theorem as the principal integrated function, developing the transects as the hypotenuses of triangles whose other sides are sides of the unit square.

The scanning scheme of this double integration is illustrated by Figure One below:-

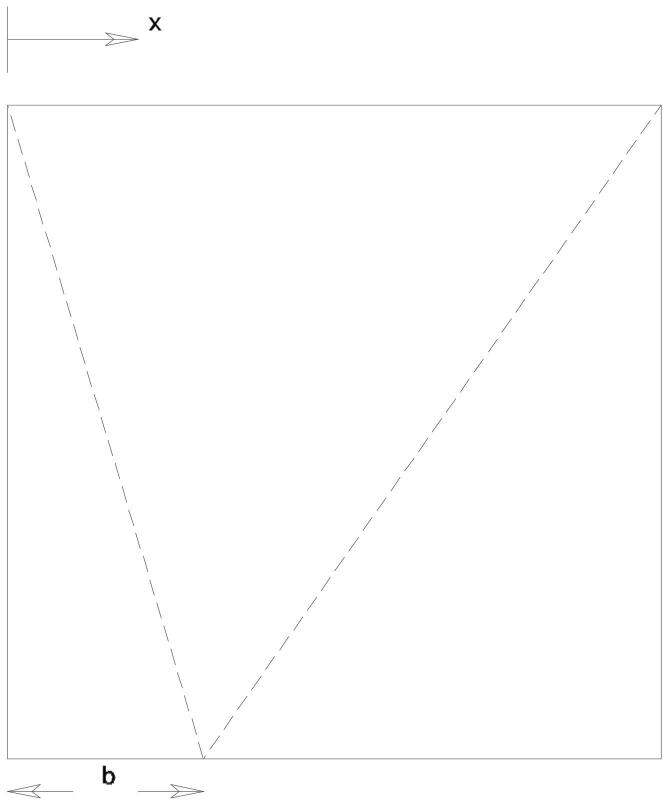


Figure 1

The Integration Scheme for
The Mean Transect of a Square

It is therefore possible to develop Equation One as:-

$$h_s^2 = \int_0^1 \int_0^1 h^2 \, dx.db \qquad \textbf{Eqn.2}$$

where h_s is either h_L, the Mean Transect Length to the Left-Hand Side; h_T the Length to the Top Side; or h_R the Length to the Right-Hand Side.

Because a transect projected from the base has equal probabilities of meeting any of the three remaining sides we may write:-

$$p(left) = p(top) = p(right) \qquad \textbf{Eqn.3}$$

and:-

$$p(left) + p(top) + p(right) = 1 \qquad \textbf{Eqn.4}$$

and hence that:-

$$h_\mu = \frac{1}{3} h_L + \frac{1}{3} h_T + \frac{1}{3} h_R = \frac{1}{3}(h_L + h_T + h_R) \qquad \textbf{Eqn.5}$$

<u>The Crossing Transection</u>

A transect from the base to the top crosses the entire height of the square to generate, with infinitely many other such examples, the length h_T.

The size of any particular h^2 is accordingly:-

$$h^2 = y^2 + (x - b)^2 \qquad \textbf{Eqn.6}$$

We may substitute these values into Equation Two to give:-

$$h_T^2 = \int_0^1 \int_0^1 y^2 + (x - b)^2 \, dx.db \qquad \textbf{Eqn.7}$$

Accordingly:-

$$h_T^2 = \int_0^1 \int_0^1 1 + x^2 - 2xb + b^2 \, dx.db$$

$$= \int_0^1 \left[x + \frac{1}{3} x^3 - b x^2 + x b^2 \right]_0^1 .db$$

$$= \int_0^1 b^2 - b + \frac{4}{3} .db$$

$$= \left[\frac{1}{3} b^3 - \frac{1}{2} b^2 + \frac{4}{3} b \right]_0^1$$

$$\therefore h_T^2 = \frac{1}{3} - \frac{1}{2} + \frac{4}{3} = \frac{2 - 3 + 8}{6} = \frac{7}{6}$$

$$\therefore h_T^2 = 1.1666666'$$

Taking the root the mean crossing transect length is:-

$$h_T = 1.08012344973$$

Using four thousand pairs of random numbers generated by EXCEL to construct an empirical h_T I obtained $h_{T,emp}$ = 1.076269, itself the mean of twelve recalculative experiments.

Let us define the Percentage Error, ε, as:-

$$\varepsilon = 100 \left(\frac{h_T - h_{T,emp}}{h_{T,emp}} \right) \qquad \textbf{Eqn.8}$$

then:-

$$\varepsilon = 100 \left(\frac{1.080123 - 1.076269}{1.076269} \right) \qquad \textbf{Eqn.9}$$

$$= 0.358162\%$$

I do not know the source of this small error though I suspect systematic biases in my worksheet scheme.

<u>The Left-Hand Transection</u>

A transect from the base to the left-hand side may have any altitude and a length between zero and the square root of two. Infinitely many examples have the mean length h_L.

The size of any particular h^2 is accordingly:-

$$h^2 = x^2 + (xb)^2 \qquad \textbf{Eqn.10}$$

taking x to vary along the left-hand side in this context. Readers should note that for any b we integrate an infinite nest of *parallel* not *radial* lines.

We may substitute these values into Equation Two to give:-

$$h_L^2 = \int_0^1 \int_0^1 x^2 + x^2 b^2 \, dx.db \qquad \textbf{Eqn.11}$$

Accordingly:-

$$h_L^2 = \int_0^1 \int_0^1 x^2 + x^2 b^2 \, dx.db$$

$$= \int_0^1 \left[\frac{1}{3} x^3 + \frac{1}{3} x^3 b^2 \right]_0^1 .db$$

$$= \int_0^1 \frac{1}{3} + \frac{1}{3} b^2 .db$$

$$= \left[\frac{1}{3} b + \frac{1}{4} b^3 \right]_0^1$$

$$\therefore h_L^2 = \frac{1}{3} + \frac{1}{4} = \frac{4+3}{12} = \frac{7}{12}$$

$$\therefore h_L^2 = 0.58333333'$$

Taking the root the mean left-projecting transect length is:-

$$h_L = 0.76376261582$$

If for values of b in the range b=0,0.1,...,1 a researcher generates a Simpsonian Integral of x^2+b^2 for the intervals of integration x=0,0.1,...,1 then he acquires a set of eleven mean transects for each b value and these are $h_{L,b}$. Feeding these transects into a twelfth integration produces an empirical $h_{L,emp}$ = 0.765225 (using EXCEL worksheets).

A second worksheet experiment generated four thousand pairs of random numbers on the interval (0,1). These then defined the opposite and adjacent of a Pythagorean triangle whose hypotenuse was a specimen transect. The four thousand transects were then averaged and ten trials of the sheet averaged to give $h_{L,emp}$ = 0.766428.

The mean of these two experimental composites is 0.7658265. When this figure is inserted into Equation Eight with the analytic mean transect it is determined that the relative error is -0.269498%, which I ascribe to computer error.

The Right-Hand Transection

A transect from the base to the right-hand side may also have any altitude and a length between zero and the square root of two. Infinitely many examples have the mean length h_R.

We may appeal to symmetry arguments to establish the identity of h_L and h_R.

Imagine that the swarms of specimen transects develop a vertical transparent film suspended in mid-air. Walk to the other side of this picture to see the projections upon the left-hand side of the square form projections upon the right, conserving all linear and angular magnitudes.

We now have an appreciation of the identity of the lateral mean transects by appeal to a system comparable to that of the Euclidean *pons asinorum*.

The Mean Transect

By reference to Equation Five we may now develop the mean transect as:-

$$h_\mu = \frac{1}{3}(h_L + h_R + h_T)$$
$$= \frac{1}{3}\left(\sqrt{\frac{7}{12}} + \sqrt{\frac{7}{12}} + \sqrt{\frac{7}{6}}\right)$$

therefore:-

$$h_\mu = \frac{1}{3}\left(2\sqrt{\frac{7}{12}} + \sqrt{\frac{7}{6}}\right) \qquad \textbf{Eqn.12}$$

or:-

$$h_\mu = 0.86921622712$$

PART II

NUMERICAL APPROACHES

Numerical approaches to the mean transect also involve a double integration in the b and x variables of different sides. As with the square, it is possible to abridge the computational process by appeals to symmetry.

Such a double integration constitutes a numerical cubature based upon the expectation of transect length expressed by:-

$$E(t) = \int_0^1\int_0^1 t.dx.db \qquad \textbf{Eqn.13}$$

It is possible to employ Carnot's Theorem (The Cosine Rule) to express t in terms of b, x and polygon parameters including The Side Serial Number, η. This latter parameter is defined with respect to the arbitrary basal side along which b elapses.

Appendix One entitled The Trigonometry of An Arbitrary Transect of a Heptagon illustrates the general geometrical relationship of a transect to its polygon.

The Semi-Internal Angle (The Angle Subtended by The Vertex Radius) ψ, is given by:-

$$\psi = \pi\left(\frac{1}{2} - \frac{1}{n}\right) \qquad \textbf{Eqn.14}$$

from which the auxiliary functions:-

$$u = \mathrm{Cos}(\psi) \qquad \textbf{Eqn.15}$$
$$v = \mathrm{Sin}(\psi) \qquad \textbf{Eqn.16}$$

may be defined. Because the polygon has unit sides The Vertex Radius, r, may be defined as:-

$$r = \frac{1}{2u} \qquad \textbf{Eqn.17}$$

whilst Polygon Area, A, (not required for solution) may be expressed by:-

$$A = n\left(\frac{\sqrt{r^2 - \frac{1}{4}}}{2}\right) \qquad \textbf{Eqn.18}$$

Reference to the diagram betrays the following Carnot relationships where θ is the (central) Angle Subtending Transect t:-

$$t^2 = s^2 + y^2 - 2sy\,\mathrm{Cos}\,\theta \qquad \textbf{Eqn.19}$$

where:-

$$s^2 = r^2 + b^2 - 2rb\,\mathrm{Cos}\,\psi \qquad \textbf{Eqn.20}$$

$$y^2 = r^2 + x^2 - 2rx\,\mathrm{Cos}\,\psi \qquad \textbf{Eqn.21}$$

Furthermore, the Intervertex Angle, ω, (not marked) is given by:-

$$\omega = (\eta + 1).\frac{2\pi}{n} \qquad \textbf{Eqn.22}$$

whilst:-

$$\omega = \theta + \beta + \chi \qquad \textbf{Eqn.23}$$

where β and χ are Flanking Angles. Therefore:-

$$\theta = \omega - \beta - \chi \qquad \textbf{Eqn.24}$$

Now The Sine Rule can be used to define β and χ in terms of b, x and parameters thus:-

$$\beta = \mathrm{Sin}^{-1}\left(\frac{\mathrm{b}}{\mathrm{s}}\mathrm{Sin}\,\psi\right) \qquad \textbf{Eqn.25}$$

$$\chi = \mathrm{Sin}^{-1}\left(\frac{\mathrm{x}}{\mathrm{y}}\mathrm{Sin}\,\psi\right) \qquad \textbf{Eqn.26}$$

The basal side is of η=0 and moving sinistrally the next is η=1,etc.
From this position appropriate substitutions allow us to define t as:-

$$t = \sqrt{\{2r[r - u(b + x)] + b^2 + x^2\} - 2\sqrt{r^2 + b^2 - 2rbu}\,\sqrt{r^2 + x^2 - 2rxu}\,.\mathrm{Cos}\,\theta}$$

Eqn.27

where:-

$$\theta = (\eta + 1).\frac{2\pi}{n} - \mathrm{Sin}^{-1}\left(\frac{\mathrm{b}}{\sqrt{\mathrm{r}^2 + \mathrm{b}^2 - 2\,\mathrm{rbu}}}.\mathrm{v}\right) - \mathrm{Sin}^{-1}\left(\frac{\mathrm{x}}{\sqrt{\mathrm{r}^2 + \mathrm{x}^2 - 2\,\mathrm{rxu}}}.\mathrm{v}\right)$$

Eqn.28

Given n and η, it is clear that for any b s and β are constant. During integration with respect to x we can therefore exploit the following simplification:-

$$t = \{2r[r - u(b + x)] + b^2 + x^2\} - 2s\sqrt{r^2 + x^2 - 2rxu}.C$$ **Eqn.29**

where:-

$$C = \mathrm{Cos}\left[\omega - \beta - \mathrm{Sin}^{-1}\left(\frac{\mathrm{x}}{\sqrt{\mathrm{r}^2 + \mathrm{x}^2 - 2\,\mathrm{rxu}}}.\mathrm{v}\right)\right]$$

Now as aforementioned, symmetry considerations allow us to establish the mean transect using only half (say the right-hand half) of the remaining sides in the case of odd-sided polygons. In the case of even-sided polygons we integrate on the right-hand sides and also the "top" side opposite and parallel to the basal side. Allow the mean transect for a particular side η of a particular polygon n to be $t_{\eta,n}$. Then:-

$$t_{\mu,n} = \int_0^1\int_0^1 t.dx.db$$ **Eqn.30**

Define the Side Count Auxiliaries j, k and m as:-

$$j = \frac{1 + (-1)^n}{2}$$ **Eqn.31**

$$k = \frac{1 - (-1)^n}{2}$$ **Eqn.32**

$$m = \frac{n + k}{2}$$ **Eqn.33**

where m is the Number of Sides to be Integrated.

Then Mean Transect for The Polygon of n-Sides, T_n, is given by:-

$$T_n = \frac{1}{n - 1}\left(2\sum_{\eta=1}^{m-1} t_{\eta,n} + jt_{m,n}\right)$$ **Eqn.34**

<u>The Choice of The Numerical Integrator</u>

Two fundamental genuses of numerical integration algorithms were considered:-

(a) Romberg Methods

(b) Newton-Cotes Formulae

Several varieties of each were tested in the QBASIC program TRANSECT.OLD (not appended to this paper).

A test integral was used in the form of:-

$$I = \int_{0.76}^{0.8378} \int_{0.435}^{1.289} x^2 + b^3 . dx.db \quad \textbf{Eqn.35}$$

whose partial solution with respect to x is:-

$$ps_x = 0.68646256466 + 0.854\, b^3 \quad \textbf{Eqn.36}$$

giving 1.06134806866 when b=0.76, which is the solution of:-

$$I_x = \int_{0.435}^{1.289} x^2 + 0.438976\ .dx \quad \textbf{Eqn.37}$$

and this latter form was used in preliminary method assessments.
The methods tested are classified as below:-

A.1 Warren's Romberg Algorithm
(Subroutine ROMBERG)

This drives to an accuracy defined by the number of trapezoidal integrations to be performed. Though excellent in contexts such as the computation of Gaussian probabilities it gave only three-figure accuracy for this application and was abandoned at the outset.

A.2 Scraton's Method[1]
(Subroutine ROMBERG!)

This QBASIC adaptation of Scraton's Romberg algorithm used an integer pass counter. Better than four-figure accuracy could not be achieved without overflow.

A.3 Sprott's Process[2]
(Subroutines QROMB, TRAPZD, POLINT)

This subjectively appeared swift returning something like ten-figure accuracy.

Newton-Cotes formulae were concatenated for forty-eight intervals of the range of integration. This gave 24 Simpson phases, 8 Weddle phases and 6 Rule Eighteen phases.

The forty-nine required ordinates were pre-loaded into an array called WA().

B.1 Simpson's Rule
(Subroutine SIMPSON1)
This gave some five-figure accuracy.

B.2 Weddle's Rule
(Subroutine WEDDLE)
This gave approximately eight-figure accuracy.

B.3 Abramowitz and Stegun Rule Eighteen[3]
(Subroutine RULE18)

This gave at least eleven-figure accuracy agreeing precisely with the analytic solution and was accordingly adopted. The basic phase of this Newton-Cotes formula is presented as Rule 25.4.18 on Page 886 of Abramowitz and Stegun.

The POLYGON Cubature Engine

The QBASIC routine used to compute mean transects by numerical cubature is called TRANANAL.BAS and is given in Appendix Two.

The required double integration of Equation Thirty is implemented as Subroutine POLYGON which embodies NA+1 applications of Subroutine RULE18, a numerical quadrature in x for a series of NA+1 b ordinates. These b ordinates are then integrated by RULE18 to yield the double integration.

This cubature is performed for the m necessary and sufficient polygon sides and Equation Thirty-Four applied to yield the Mean Transect as element TT(N).

Accordingly the cubature results from NA+2 quadratures where NA is the Number of Integration Intervals. Tests indicated that four phases of Rule 18 (i.e. NA=32) gave eleven-figure quadrature accuracy with the test polynomial but this degenerates to seven-figure accuracy when cubature is completed on the transect function of Equation Twenty-Nine.

Comparative Results

In this discussion the Part One Error Equation, Equation Eight will be adopted to indicate relative errors between TRANANAL.BAS mean transects and those determined by other means. If T_A is a TRANANAL.BAS Mean Transect and T_O a Reference Mean Transect then:-

$$\varepsilon = 100\left(\frac{T_A - T_O}{T_O}\right) \qquad \textbf{Eqn.38}$$

Since for the square the TRANANAL mean is 0.869009129786 and the analytic integral 0.86921622712 the error of the two methods is -0.0238257563%. I do not know the source of this discrepancy.

A more general comparison was performed with reference to MATHCAD integrals. MATHCAD would not, however, yield a cubature for n=3 (the triangle) or for the n=50 or n=100 cases.

The transects and their relative errors are listed in the spreadsheet TRANANCO.CAL of Appendix Three. Study of this table shows that error improves as the length variability of the $(NA+1)^2$ specimen transects diminishes, an eventuality to be expected.

At -0.000011% the error is worst for the square, and at -0.00000004% it is best for the n=100 hectogon. The mean error is -0.000002028811874. This is precise agreement and the two cubature engines are as good as one another in this application. I do not know which mechanism is employed for MATHCAD integrations.

PART III

STATISTICAL APPROACHES

Statistical transect generation is based upon the computation of two arbitrary angles proportional to two random numbers such that the interval 0,1 maps to the interval 0,2π. For each (central) angle, the Cartesian co-ordinates of the intersection of its radius with the appropriate polygon side are calculated and Pythagoras Theorem used to determine the specimen transect traced.

An adequate number of such transects are determined and their lengths averaged.

This process is programmed in TRANSECT.BAS of Appendix Four. The given program is configured for 64 trials but the table in Appendix Five shows that the 4096-trial experiment was the most reliable.

Those who read my paper entitled "An Evaluation of the Efficiency of Some Pseudo-Random Number Generators" may recall that I was broadly unhappy with the routines published by Sprott[2] and determined that for general purposes the QBASIC intrinsic function RND() was superior to any.

Nevertheless I used Sprott's RAN2 subroutine in TRANSECT.BAS because of the closeness of its generated mean number to 0.5 when it was applied with a seed of 466513 to 4096 trials. RAN2 comprises a single congruential generator and a shuffler to return a uniform random deviate.

As configured for 4096 trials, TRANSECT.BAS took about two and a half hours to compute as opposed to around half an hour for TRANANAL.BAS. Both programs were *interpreted*.

Comparative Results

Appendix Six presents spreadsheet TRANANCP.CAL, which tabulates 4096-trial TRANSECT.BAS mean transects against TRANANAL.BAS mean transects and their relative errors.

Triangle mean transects are highly anomalous. That due to numerical cubature is about 0.715096 whereas that due to statistical trials is about 0.573990. After many attempts I failed to determine an analytic mean transect for the triangle and so am unable to arbitrate.

Readers can see that excepting that n=3 case the relative absolute error is less than 1% for all the polygons tested. If the triangle is included the mean absolute error is about 2.43% and if the triangle is excluded it is about 0.41%.

PART IV

CONCLUSIONS

Analytic determinations of the mean transect are possible only for the square, but arbitrarily precise numerical estimates are available for all regular polygons given a sufficiently stable integration engine.

Cubature is practicable by quadrature of each ordinate of the first independent variable (x) followed by a single summative quadrature with respect to the second (b). An adjusted summation of the cubatures for non-redundant polygon sides will then yield the mean transect.

Numerical determinations are utterly superior to statistical ones with respect both to speed and to accuracy.

Numerical and statistical determinations of the triangle mean transect are not in agreement.

Notation

A	The Polygon Area
β	The Basal Flanking Angle
b	The Basal Variable
C	The Cosine of the Angle Subtending The Transect
db	The Infinitesimal of b
dx	The Infinitesimal of x
ε	The Relative Percentage Error
E(x)	The Expectation of x
E(t)	The Expectation of Transect Length
f(x)	The Function of x
η	The Polygon Side Serial Number
h	A Transect (Length)
h_μ	The Mean Transect Length
h_s	The Mean Transect Length to a Side
h_L	The Mean Transect Length to the Left Side
$h_{L,b}$	A Left-Side Transect for a Given b Value
$h_{L,emp}$	A Statistical Estimate of h_L
h_R	The Mean Transect Length to the Right Side
h_T	The Mean Transect Length to the Top Side
$h_{T,emp}$	A Statistical Estimate of h_T
θ	The Angle Subtending the Transect
I	A Definite Integral
I_x	A Definite Integral for Constant b
j	The Evenness Indicator
k	The Oddness Indicator
m	The Number of Sides to be Integrated
n	The Number of Polygon Sides
NA	The Number of Intervals of Numeric Integration
π	The Ludolphine Constant
p(*)	The Probability of Condition *
ps_x	A Partial Solution with Respect to x
r	The Vertex Radius
s	The Basal Radius
t	A Transect Length

$t_{\eta,n}$	The Mean Transect of Side η Only
$t_{m,n}$	The Mean Transect of the Final Side Only
T_n	The Mean Transect of an n-Sided Polygon
T_A	A (Numeric) Mean Transect due to TRANANAL.BAS
T_O	A Mean Transect Computed by Other Means
u	The Cosine of ψ
v	The Sine of ψ
χ	The Projected Flanking Angle
x	The Independent Variable
	The Projected Variable
ψ	The Semi-Interval Angle
	(The Angle Subtended by the Vertex Radius)
y	The Square Height (unity)
	The Projected Radius
ω	The Intervertex Angle

References

1 "Basic Numerical Methods"
RE Scraton 1984
Edward Arnold of London
ISBN 0-7131-3521-2

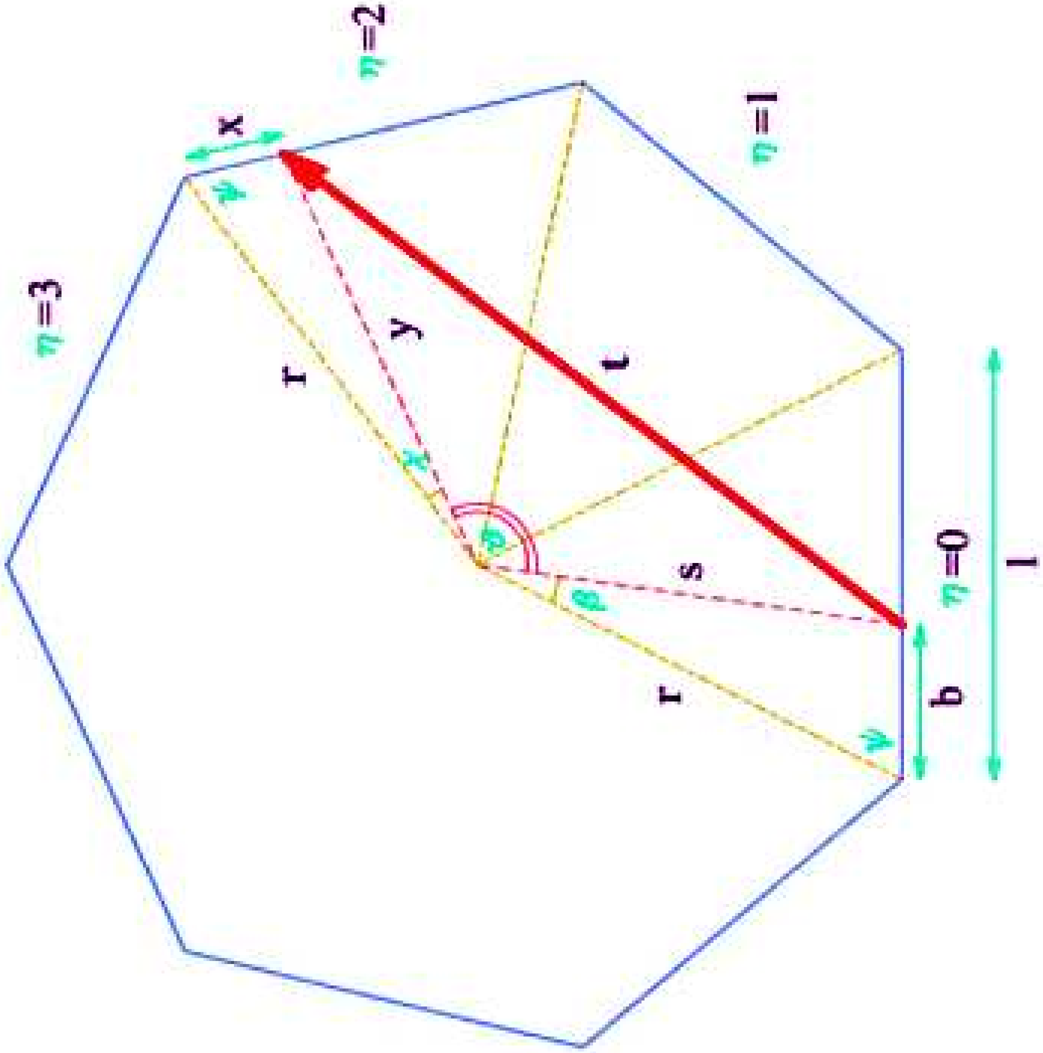
2 "Numerical Recipes Routines and Examples in BASIC"
Julien C Sprott 1991
Cambridge University Press
ISBN 0-521-40689-7

3 "Handbook of Mathematical Functions"
Milton Abramowitz and Irene A Stegun 1965
Dover Publications of New York
ISBN 0-486-61272-4

Appendix One

The Trigonometry of an Arbitrary Transect of a Heptagon

File: HEPTAGONSatVert.gif


η=3
η=2
x
η=1
y
r
t
s
r
η=0
b
l

Appendix Two

Program TRANANAL.BAS

```
'     PROGRAM TRANANAL.BAS
'     A PROGRAM TO COMPUTE THE MEAN TRANSECT LENGTH
'     FOR A SERIES OF REGULAR POLYGONS
'     OF UNIT SIDE LENGTH
'     BY THE NUMERIC DOUBLE INTEGRATION OF
'     AN ANALYTIC EXPRESSION FOR THE TRANSECT
'
'     THE OUTPUT FILE BEARS THE EXTENSION SHOWN:-
'
'         THE STATISTICS OUTPUT FILE      .TRL
'
'     WRITTEN BY:-
'
'       JAMES R WARREN BSc MSc PhD PGCE MIEE(CS) MACM
'       "SOUTHGATE"
'       31 VICTORIA AVENUE
'       BLOXWICH
'       WS3 3HS
'       UNITED KINGDOM
'
'       2 JANUARY 1998
'
'     THIS PROGRAM IS WRITTEN IN MICROSOFT QBASIC
'
' VARIABLE TYPE DEFAULTS
      DEFDBL A-H, O-R, T-Z
      DEFSTR S
      DEFINT I-K, M-N
      DEFLNG L
' SEGMENT DECLARATIONS
      DECLARE SUB AREA (I, N, PH, U, V, R, RO, AR())
      DECLARE FUNCTION ASIN (A)
      DECLARE SUB BXCREATE (NA, B(), B2(), X(), X2())
      DECLARE SUB DATANAL (NG, N(), TT(), AR())
      DECLARE SUB FINALE ()
      DECLARE SUB NOTE (S)
      DECLARE SUB POLYGON (I, N, U, V, R, RO, OM, NP, NA, IE, B(), B2(), X(), X2(),
AR(), WA(), T(), TS(), TT())
      DECLARE SUB RULE18 (NP, FLB, FUB, WA(), RI)
      DECLARE FUNCTION TSQR (X)
      DECLARE SUB WAFILL (N, U, V, R, RO, OM, NA, J, IE, B(), B2(), X(), X2(), WA())
      DECLARE SUB WARBLE ()
' COMMON VARIABLES
      COMMON SHARED IA, SA
      COMMON SHARED PI, HP, PI2
      COMMON SHARED SC, SM, SCR
      COMMON SHARED IU, IV, SP, SF, SXV, SXW
' STATIC ARRAY DEFINITIONS
      DIM N(20)
      DIM B(512), B2(512), X(512), X2(512), WA(512), T(512)
      DIM TS(200)
```

```
      DIM TT(20), AR(20)
' DYNAMIC ARRAY LIMIT
'     ( none )
' DYNAMIC ARRAY DEFINITIONS
'     ( none )
' DEVICE ATTRIBUTIONS
      SCREEN 12: WINDOW (1, 1)-(640, 480)
' LOGICAL UNIT, EXTENSION AND PATHNAME SETTINGS
      IU = 1: IV = 2
      SXV = ".CSV": SXW = ".TRL"
      SP = "C:\QBASIC\QBFILES\"
' FORMAT DEFINITIONS
'     ( none )
' NUMERICAL CONSTANT DEFINITIONS
      PI = 3.141592653589793#
      PI2 = 2# * PI
      HP = PI / 2#
' STRING CONSTANT DEFINITIONS
      SC = ":": SM = ",": SCR = CHR$(13) + CHR$(10)
' TEXT VARIABLE DEFINITIONS
'     ( none )
'
' ** THE ALGORITHM **
'
      IB = 8
      NP = 4
      NA = IB * NP
      NG = 12
      FOR I = 1 TO NG: READ N(I): NEXT I
      DATA 3,4,5,6,7,8,9,10,15,20,50,100
      BXCREATE NA, B(), B2(), X(), X2()
      FOR I = 1 TO NG
        AREA I, N(I), PH, U, V, R, RO, AR()
        POLYGON I, N(I), U, V, R, RO, OM, NP, NA, IE, B(), B2(), X(), X2(), AR(), WA(),
T(), TS(), TT()
      NEXT I
      DATANAL NG, N(), TT(), AR()
      FINALE
      END

      SUB AREA (I, N, PH, U, V, R, RO, AR())
' A SUBROUTINE TO COMPUTE THE AREA OF A POLYGON TOGETHER WITH
' OTHER POLYGON STATISTICS
'     ARGUMENTS:
'       I     THE POLYGON SERIAL NUMBER
'       N     THE NUMBER OF POLYGON SIDES
'       PH    THE ANGLE SUBTENDED BY THE VERTEX RADIUS
'       U     THE COSINE OF PH
'       V     THE SINE   OF PH
'       R     THE VERTEX RADIUS
'       RO    THE SQUARE OF THE VERTEX RADIUS
```

```
'         AR()   THE ARRAY OF POLYGON AREAS
' ( PI IS COMMON SHARED )
'
     PH = PI * (.5# - 1 / N): U = COS(PH): V = SIN(PH)
     R = 1 / (2 * U): RO = R * R: AR(I) = N * (SQR(RO - .25#) / 2)
     END SUB

     FUNCTION ASIN (A)
' A FUNCTION TO COMPUTE THE INVERSE SINE OF ANGLE A
' ( HP IS COMMON SHARED )
     IF A = 1 THEN
         ASIN = HP
       ELSE
         ASIN = ATN(A / SQR(1 - A * A))
     END IF
     END FUNCTION

     SUB BXCREATE (NA, B(), B2(), X(), X2())
' A SUBROUTINE TO FILL THE DOUBLE INTEGRATION ARRAYS
'      ARGUMENTS:
'         NA     THE NUMBER OF INTERVALS
'         B()    THE ARRAY OF B VALUES
'         B2()   THE ARRAY OF B SQUARED VALUES
'         X()    THE ARRAY OF X VALUES
'         X2()   THE ARRAY OF X SQUARED VALUES
'
     FOR I = 1 TO NA + 1
       J = I - 1: B(I) = J / NA: X(I) = B(I): B2(I) = B(I) * B(I): X2(I) = B2(I)
     NEXT I
     END SUB

     SUB DATANAL (NG, N(), TT(), AR())
' A SUBROUTINE TO WRITE MEAN TRANSPUT STATISTICS TO FILE
TRANSECT.TRA
'      ARGUMENTS:
'         NG     THE NUMBER OF POLYGONS
'         N()    THE ARRAY OF POLYGON NUMBERS OF SIDES
'         TT()   THE ARRAY OF MEAN TRANSECTS
'         AR()   THE ARRAY OF POLYGON AREAS
' ( IV, SP, SXW AND SM ARE COMMON SHARED )
'
     OPEN "O", IV, SP + "TRANSECT" + SXW
     PRINT #IV, "THE MEAN TRANSECT OF A REGULAR POLYGON"
     PRINT #IV, "ESTIMATED BY THE NUMERICAL DOUBLE INTEGRATION"
     PRINT #IV, "OF AN ANALYTIC EXPRESSION FOR THE TRANSECT"
     PRINT #IV,
     PRINT #IV, "TRANSECT.TRL"
     PRINT #IV,
     PRINT #IV, "NUMBER"; SM; "MEAN"; SM; "POLYGON"
     PRINT #IV, "OF"; SM; "TRANSECT"; SM; "AREA"
     PRINT #IV, "SIDES"
```

```
      PRINT #IV,
      FOR I = 1 TO NG
        PRINT #IV, N(I); SM; TT(I); SM; AR(I)
      NEXT I
      PRINT #IV,
      PRINT #IV,
      PRINT #IV, "ALL STATISTICS ARE DETERMINED FOR THE UNIT SIDE"
      CLOSE IV
      END SUB

      SUB FINALE
' A SUBROUTINE TO SOUND THE PROGRAM TERMINATION SIGNAL
'
      NOTE "3C0506"
      NOTE "4D0509"
      NOTE "5E0512"
      NOTE "6F0515"
      END SUB

      SUB NOTE (S)
' A SUBROUTINE TO SOUND A NOTE UPON THE COMPUTER SPEAKER
'     ARGUMENT:
'        S     THE NOTE SPECIFIER STRING "IN$NL" e.g. "2B0506"
'              I   THE OCTAVE NUMBER        ( 0-6 )
'              N$  THE NOTE LETTER          ( ABCDEFG )
'              N   THE NOTE NUMBER          ( 0-84 )
'              L   THE LENGTH OF THE NOTE   ( 1-64 )
'
      I = VAL(MID$(S, 1, 1)): N$ = MID$(S, 2, 1)
      N = VAL(MID$(S, 3, 2)): L = VAL(MID$(S, 5, 2))
      PLAY "O" + STR$(I) + "N" + STR$(N) + "L" + STR$(L) + "X" + VARPTR$(N$)
      END SUB

      SUB POLYGON (I, N, U, V, R, RO, OM, NP, NA, IE, B(), B2(), X(), X2(), AR(),
WA(), T(), TS(), TT())
' A SUBROUTINE TO DETERMINE THE MEAN TRANSECT OF A POLYGON BY
NUMERIC
' DOUBLE INTEGRATION EMPLOYING GENERATED ANALYTIC TRANSECT
VALUES
'     ARGUMENTS:
'        I      THE POLYGON SERIAL NUMBER
'        N      THE NUMBER OF POLYGON SIDES
'        U      THE COSINE OF PH
'        V      THE SINE   OF PH
'        R      THE VERTEX RADIUS
'        RO     THE SQUARE OF THE VERTEX RADIUS
'        OM     THE RADIAL DISPLACEMENT ANGLE
'        NP     THE NUMBER OF TERM CYCLE PHASES
'        NA     THE EXTENT OF ARRAY WA()
'        IE     THE POLYGON SIDE SERIAL NUMBER
'        B()    THE ARRAY OF B VALUES
```

```
'        B2()   THE ARRAY OF B SQUARED VALUES
'        X()    THE ARRAY OF X VALUES
'        X2()   THE ARRAY OF X SQUARED VALUES
'        AR()   THE ARRAY OF POLYGON AREAS
'        WA()   THE ARRAY OF TRANSECT FUNCTIONS
'        T()    THE ARRAY OF B-INTEGRALS IN X
'        TS()   THE ARRAY OF SIDE INTEGRALS
'        TT()   THE ARRAY OF MEAN TRANSECTS
' ( PI AND PI2 ARE COMMON SHARED )
'
      M = (N / 2) + (1 / 2) * (1 - (-1) ^ N) / 2
      FOR IE = 1 TO M
        FOR J = 1 TO NA + 1
          WAFILL N, U, V, R, RO, OM, NA, J, IE, B(), B2(), X(), X2(), WA()
          RULE18 NP, 0#, 1#, WA(), T(J)
        NEXT J
        RULE18 NP, 0#, 1#, T(), TS(IE)
      NEXT IE
      TM = 0#
      FOR IE = 1 TO M - 1
        TM = TM + TS(IE)
      NEXT IE
      TT(I) = (1 / (N - 1)) * (2 * TM + TS(M) * (1 + (-1) ^ N) / 2)
      END SUB

      SUB RULE18 (NP, FLB, FUB, WA(), RI)
' A SUBROUTINE TO COMPUTE AN NP-PHASE NUMERICAL INTEGRATION USING A CLOSED TYPE
' NEWTON-COTES FORMULA BASED UPON EIGHT INTERVALS AND CITED AS RULE 25.4.18 ON
' PAGE 886 OF "HANDBOOK OF MATHEMATICAL FUNCTIONS" EDITED BY MILTON ABRAMOWITZ
' AND IRENE A STEGUN, DOVER PUBLICATIONS OF NEW YORK 1965,ISBN 0-486-61272-4.
' THE REQUISITE FUNCTION ORDINATES ARE PRE-ASSIGNED TO WA(1) THROUGH WA(1+NP*8).
' ERROR IS OF ORDER H^11.
'     ARGUMENTS:
'        NP     THE NUMBER OF TERM CYCLE PHASES
'        FLB    THE FUNCTION LOWER BOUND
'        FUB    THE FUNCTION UPPER BOUND
'        WA()   THE ARRAY OF TRANSECT FUNCTIONS
'        RI     THE RULE 18 INTEGRAL
'
      IB = 8
      H = (FUB - FLB) / (IB * NP): D = 2.821869488536155D-04 * H: IL = 1 + IB * NP
      DIM IY(IB)
      IY(0) = 5888: IY(1) = -928: IY(2) = 10496: IY(3) = -4540: IY(4) = 10496
      IY(5) = -928: IY(6) = 5888: IY(7) = 1978
      T = 989 * WA(1)
      FOR I = 2 TO IL
```

```
        J = I - (2 + IB * INT((I - 2) / IB))
        T = T + IY(J) * WA(I)
      NEXT I
      T = T - WA(IL) * 989
      RI = D * T
      END SUB

      FUNCTION TSQR (X)
' A SUBROUTINE TO COMPUTE A REAL SQUARE ROOT AFTER SETTING ANY
' VERY SMALL NEGATIVE ARGUMENTS TO ZERO.
' THE TOLERENCE TL IS MEANTIME SET TO 0.00000000000001.
'
      TL = .00000000000001#
      IF X >= 0# THEN
          TSQR = SQR(X)
        ELSE
          IF X >= -TL THEN
              TSQR = 0#
            ELSE
              PRINT "NEGATIVE ARGUMENT OF SQRT"
              NOTE "3C0506"
              END
          END IF
      END IF
      END FUNCTION

      SUB WAFILL (N, U, V, R, RO, OM, NA, J, IE, B(), B2(), X(), X2(), WA())
' A SUBROUTINE TO FILL THE ARRAY WA(NA) WITH VALUES OF THE
TRANSECT FUNCTION
' AT B=BB. THIS REPRESENTS A COLUMN OF X-VARIANT INTEGRATION
ORDINATES
'     ARGUMENTS:
'        N      THE NUMBER OF POLYGON SIDES
'        U      THE COSINE OF PH
'        V      THE SINE   OF PH
'        R      THE VERTEX RADIUS
'        RO     THE SQUARE OF THE VERTEX RADIUS
'        OM     THE RADIAL DISPLACEMENT ANGLE
'        NA     THE NUMBER OF INTERVALS
'        J      THE SUBSCRIPT OF THE COLUMN B() ARRAY ELEMENT
'        IE     THE POLYGON SIDE SERIAL NUMBER
'        B()    THE ARRAY OF B VALUES
'        B2()   THE ARRAY OF B SQUARED VALUES
'        X()    THE ARRAY OF X VALUES
'        X2()   THE ARRAY OF X SQUARED VALUES
'        WA()   THE ARRAY OF TRANSECT FUNCTIONS
' ( PI AND PI2 ARE COMMON SHARED )
'
' CONSTANT FOR A PARTICULAR SIDE
      OM = (IE + 1) * PI2 / N
' CONSTANTS FOR A PARTICULAR B
```

```
      BC = SQR(RO + B2(J) - 2 * R * B(J) * U)
      BA = ASIN(B(J) * V / BC)
' FILLING WA()
      FOR I = 1 TO NA + 1
        XC = SQR(RO + X2(I) - 2 * R * X(I) * U)
        WA(I) = TSQR((2 * R * (R - U * (B(J) + X(I))) + B2(J) + X2(I)) - 2 * BC * XC *
COS(OM - BA - ASIN(X(I) * V / XC)))
      NEXT I
      END SUB

      SUB WARBLE
' A SUBROUTINE TO SOUND A SUCCESSION OF FOUR NOTE CALLS
' ON THE COMPUTER SPEAKER
      NOTE "3C0506": NOTE "4D0509": NOTE "5E0512": NOTE "6F0515"
      END SUB
```

Appendix Three

Comparative Numerical Solutions
for
The Mean Transect of Selected Polygons
(TRANANCO.CAL)

File: TRANSET1.XLS:TRANANCO

THE MEAN TRANSECT OF A REGULAR POLYGON
ESTIMATED BY THE NUMERICAL DOUBLE INTEGRATION
OF AN ANALYTIC EXPRESSION FOR THE TRANSECT

TRANANCO.TRL

NUMBER OF SIDES	MEAN TRANSECT BY TRANANAL.BAS	MEAN TRANSECT BY MATHACAD	POLYGON AREA BY TRANANAL.BAS	PERCENTAGE ERROR IN TRANSECTS	
3	0.715096396668		0.433012701892		
4	0.869009129786	0.869009226068	1.000000000000	-0.000011079498140	
5	1.104867617810	1.104867636258	1.720477400590	-0.000001669673291	
6	1.328515670260	1.328515675508	2.598076211350	-0.000000395016786	
7	1.545559576520	1.545559609848	3.633912444000	-0.000002156388515	
8	1.758666113360	1.758666136910	4.828427124750	-0.000001339058027	
9	1.969246196830	1.969246213954	6.181824193770	-0.000000869549983	
10	2.178111850140	2.178111862918	7.694208842940	-0.000000586637448	
15	3.209317720030	3.209317723996	17.642362910500	-0.000000123580158	
20	4.231147617070	4.231147618758	31.568757573400	-0.000000039904521	
50	10.325296618900		198.681810548000		
100	20.462192771900		795.512898844000		
				-0.000002028811874	MEAN

ALL STATISTICS ARE DETERMINED FOR THE UNIT SIDE

Appendix Four

Program TRANSECT.BAS

```
'     PROGRAM TRANSECT.BAS
'     A PROGRAM TO COMPUTE THE MEAN TRANSECT LENGTH
'     AND ITS HIGHER MOMENTAL STATISTICS
'     FOR A SERIES OF REGULAR POLYGONS
'     OF UNIT SIDE LENGTH
'
'     THE OUTPUT FILE BEARS THE EXTENSION SHOWN:-
'
'          THE STATISTICS OUTPUT FILE      .TRA
'
'     WRITTEN BY:-
'
'       JAMES R WARREN BSc MSc PhD PGCE MIEE(CS) MACM
'       "SOUTHGATE"
'       31 VICTORIA AVENUE
'       BLOXWICH
'       WS3 3HS
'       UNITED KINGDOM
'
'       27 OCTOBER 1997
'
'     THIS PROGRAM IS WRITTEN IN MICROSOFT QBASIC
'
' VARIABLE TYPE DEFAULTS
      DEFDBL A-H, O-R, T-Z
      DEFSTR S
      DEFINT I-K, M-N
      DEFLNG L
' SEGMENT DECLARATIONS
      DECLARE SUB ANGLEDRESS (A)
      DECLARE SUB AREAFORM (I, N(), TH, A1, A2, X1, Y1, X2, Y2, XB(), YB(), XV(),
YV(), AK, AX, AY, AR1, AR2)
      DECLARE SUB AREAPOLY (N, X(), Y(), A)
      DECLARE SUB CARTESIAN (XD, YD, R, A, X, Y)
      DECLARE SUB DATAOUT (NG, NT, N(), V(), ES(), AR())
      DECLARE SUB FINALE ()
      DECLARE SUB MOMENTAL (N, P(), AM, VP, VS, VQ, VR)
      DECLARE SUB NOTE (S)
      DECLARE SUB POLYLOAD (N, NV, JI, JF, JB, JC, X1, Y1, X2, Y2, XB(), YB(),
XV(), YV())
      DECLARE SUB POLYPOINT (TH, GA, CD, R, A, X, Y)
      DECLARE FUNCTION RAN2 (LD)
' COMMON VARIABLES
      COMMON SHARED IA, SA
      COMMON SHARED PI, HP, PI2
      COMMON SHARED SC, SM, SCR
      COMMON SHARED IU, IV, SP, SF, SXV, SXW
' STATIC ARRAY DEFINITIONS
      DIM N(50), ES(50), V(50, 5), AR(50, 4)
      DIM XB(100), YB(100), XV(102), YV(102)
' DYNAMIC ARRAY LIMIT
```

```
      ILT = 4096
' DYNAMIC ARRAY DEFINITIONS
      REDIM T(ILT), RN1(ILT), RN2(ILT)
' DEVICE ATTRIBUTIONS
      SCREEN 12: WINDOW (1, 1)-(640, 480)
' LOGICAL UNIT, EXTENSION AND PATHNAME SETTINGS
      IU = 1: IV = 2
      SXV = ".CSV": SXW = ".TRA"
      SP = "C:\QBASIC\QBFILES\"
' FORMAT DEFINITIONS
'     ( none )
' NUMERICAL CONSTANT DEFINITIONS
      PI = 3.141592653589793#
      PI2 = 2# * PI
      HP = PI / 2#
      LD = 466513
' STRING CONSTANT DEFINITIONS
      SC = ":": SM = ",": SCR = CHR$(13) + CHR$(10)
' TEXT VARIABLE DEFINITIONS
'     ( none )
'
' ** THE ALGORITHM **
'
      B = RAN2(-1)
      AK = 150: AX = 320: AY = 240
      RO = 1 / (2 * SIN(PI / 3))
' DEFINE THE REGULAR POLYGONS TO BE ASSESSED
      NG = 12
      FOR I = 1 TO NG: READ N(I): NEXT I
      DATA 3,4,5,6,7,8,9,10,15,20,50,100
' SET THE NUMBERS OF TRIALS
      NT = 64
' FILL THE ARRAY OF PSEUDO-RANDOM NUMBERS
      FOR I = 1 TO NT: RN1(I) = RAN2(LD): RN2(I) = RAN2(LD): NEXT I
' COMPUTE POLYGON STATISTICS
      FOR I = 1 TO NG
        TH = PI2 / N(I): GA = (PI - TH) / 2: GS = SIN(GA): CD = PI - GA
        R = 1 / (2 * COS(GA)): AK = AK * RO / R
' GENERATE VERTICES
        FOR J = 1 TO N(I): AN = J * TH: CARTESIAN 0, 0, R, AN, XB(J), YB(J): NEXT J
' GENERATE AND MEASURE LINES
        AU = 0!: AV = 0!
        FOR K = 1 TO NT
          CN = (N(I) - 1) / N(I): A1 = PI2 * RN1(K): AJ = CN * PI2 * RN2(K): UN = TH *
(INT(A1 / TH) + 1): A2 = UN + AJ
          ANGLEDRESS A1
          ANGLEDRESS A2
          IF A2 < A1 THEN
               A4 = A1: A1 = A2: A2 = A4
          END IF
          POLYPOINT TH, GA, CD, R, A1, X1, Y1
```

```
        POLYPOINT TH, GA, CD, R, A2, X2, Y2
        T(K) = SQR((X2 - X1) ^ 2 + (Y2 - Y1) ^ 2)
        AREAFORM I, N(), TH, A1, A2, X1, Y1, X2, Y2, XB(), YB(), XV(), YV(), AK,
AX, AY, AR1, AR2
        AU = AU + AR1: AV = AV + AR2
      NEXT K
      MOMENTAL NT, T(), V(I, 1), V(I, 2), V(I, 3), V(I, 4), V(I, 5)
      RO = R: ES(I) = V(I, 2) / SQR(NT)
      AR(I, 1) = AU / NT: AR(I, 2) = AV / NT: AR(I, 3) = AR(I, 1) + AR(I, 2)
      PJ = PI / N(I): AR(I, 4) = N(I) * R * R * COS(PJ) * SIN(PJ)
      NEXT I
    DATAOUT NG, NT, N(), V(), ES(), AR()
    FINALE
    END

    SUB ANGLEDRESS (A)
' A SUBROUTINE TO EXPRESS AN ELAPSED ANGLE ON THE INTERVAL ZERO TO
TWO PI
' EVEN WHERE THE ANGLE EXCEEDS TWO PI
'    ARGUMENT:
'      A     THE ANGLE
' ( PI2 IS COMMON SHARED )
    IY = INT(A / PI2)
    A = A - PI2 * IY
    END SUB

    SUB AREAFORM (I, N(), TH, A1, A2, X1, Y1, X2, Y2, XB(), YB(), XV(), YV(), AK,
AX, AY, AR1, AR2)
' A SUBROUTINE TO FORM THE AREAS OF PART-POLYGONS
'    ARGUMENTS:
'      I     THE FULL-POLYGON SERIAL NUMBER
'      N()   THE ARRAY OF FULL-POLYGON VERTICES
'      TH    THE VERTEX INTERANGLE
'      A1    THE FIRST  RANDOM ANGLE
'      A2    THE SECOND RANDOM ANGLE
'      X1    THE X CO-ORDINATE OF THE INITIAL VERTEX
'      Y1    THE Y CO-ORDINATE OF THE INITIAL VERTEX
'      X2    THE X CO-ORDINATE OF THE FINAL   VERTEX
'      Y2    THE Y CO-ORDINATE OF THE FINAL   VERTEX
'      XB()  THE ARRAY OF X CO-ORDINATES OF FULL-POLYGON VERTICES
'      YB()  THE ARRAY OF Y CO-ORDINATES OF FULL-POLYGON VERTICES
'      XV()  THE ARRAY OF X CO-ORDINATES OF VERTICES
'      YV()  THE ARRAY OF Y CO-ORDINATES OF VERTICES
'      AK    THE SCALING FACTOR
'      AX    THE ABSCISSAL DISPLACEMENT
'      AY    THE ORDINAL   DISPLACEMENT
'      AR1   THE PRIOR     PART-POLYGON AREA
'      AR2   THE POSTERIOR PART-POLYGON AREA
'
    I1 = INT(A1 / TH): I2 = INT(A2 / TH)
    POLYLOAD N(I), NV, 1, 1, I1 + 1, I2, X1, Y1, X2, Y2, XB(), YB(), XV(), YV()
```

```
      AREAPOLY NV, XV(), YV(), AR1
      POLYLOAD N(I), NV, 1, 0, I2 + 1, N(I), X2, Y2, X1, Y1, XB(), YB(), XV(), YV()
      POLYLOAD N(I), NV, 0, 1, 1, I1, X2, Y2, X1, Y1, XB(), YB(), XV(), YV()
      AREAPOLY NV, XV(), YV(), AR2
      END SUB

      SUB AREAPOLY (N, X(), Y(), A)
' A SUBROUTINE TO COMPUTE THE AREA OF AN ARBITRARY POLYGON USING
' THE SERIAL CARTESIAN CO-ORDINATES DEFINING ITS VERTICES
'     ARGUMENTS:
'        N      THE NUMBER OF SIDES
'        X()    THE ARRAY OF X CO-ORDINATES
'        Y()    THE ARRAY OF Y CO-ORDINATES
'        A      THE AREA
'
      A = 0: B = 0: C = 0
      FOR I = 1 TO N - 1
        B = B + X(I) * Y(I + 1): C = C - Y(I) * X(I + 1)
      NEXT I
      B = B + X(N) * Y(1): C = C - Y(N) * X(1)
      A = ABS((B + C) / 2)
      END SUB

      SUB CARTESIAN (XD, YD, R, A, X, Y)
' A SUBROUTINE TO FURNISH THE CARTESIAN CO-ORDINATES OF A POINT
DEFINED
' IN POLAR TERMS
'     ARGUMENTS:
'        XD     THE X CO-ORDINATE OF THE CENTER OF ROTATION
'        YD     THE Y CO-ORDINATE OF THE CENTER OF ROTATION
'        R      THE POLAR RADIUS
'        A      THE POLAR ANGLE
'        X      THE X CO-ORDINATE OF THE POINT
'        Y      THE Y CO-ORDINATE OF THE POINT
'
' COMPUTE CO-ORDINATES
      X = XD + R * COS(A): Y = YD + R * SIN(A)
' RETURN TO MASTER SEGMENT
      END SUB

      SUB DATAOUT (NG, NT, N(), V(), ES(), AR())
' A SUBROUTINE TO WRITE MEAN TRANSPUT STATISTICS TO FILE
TRANSECT.TRA
'     ARGUMENTS:
'        NG     THE NUMBER OF POLYGONS
'        NT     THE NUMBERS OF TRIALS
'        N()    THE ARRAY OF POLYGON NUMBERS OF SIDES
'        V()    THE ARRAY OF TRANSECT LENGTH MOMENTAL STATISTICS
'                 V(I,1)   THE MEAN
'                 V(I,2)   THE POPULATION STANDARD DEVIATION
'                 V(I,4)   THE SKEWNESS
```

```
'               V(I,5)   THE KURTOSIS
'        ES()   THE ARRAY OF STANDARD ERRORS
'        AR()   THE ARRAY OF AREA STATISTICS
'               AR(I,1)  THE PRIOR AREA
'               AR(I,2)  THE POSTERIOR AREA
'               AR(I,3)  THE SUM OF PRIOR AND POSTERIOR AREAS
'               AR(I,4)  THE ANALYTIC AREA OF THE POLYGON
' ( IV, SP, SXW AND SM ARE COMMON SHARED )
'
     OPEN "O", IV, SP + "TRANSECT" + SXW
     PRINT #IV, "THE MEAN TRANSECT OF A REGULAR POLYGON"
     PRINT #IV, "ESTIMATED BY COMPUTER AVERAGING OF"
     PRINT #IV, "INSCRIBED LINES"
     PRINT #IV,
     PRINT #IV, "TRANSECT.TRA"
     PRINT #IV,
     PRINT #IV, "THE NUMBER OF TRIALS FOR EACH POLYGON IS "; SM; NT
     PRINT #IV,
     PRINT #IV, "NUMBER"; SM; "MOMENTAL STATISTICS"; SM; SM; SM; SM; SM;
"AREA STATISTICS ( MEANS )"
     PRINT #IV, "OF"
     PRINT #IV, "SIDES"; SM; "MEAN"; SM; "POPULATION"; SM; "SKEWNESS"; SM;
"KURTOSIS";
     PRINT #IV, SM; "SE"; SM; "AREA1"; SM; "AREA2"; SM; "TOTAL"; SM;
"ANALYTIC"
     PRINT #IV, SM; SM; "SD"
     PRINT #IV,
     FOR I = 1 TO NG
       PRINT #IV, N(I); SM; V(I, 1); SM; V(I, 2); SM; V(I, 4); SM; V(I, 5);
       PRINT #IV, SM; ES(I); SM; AR(I, 1); SM; AR(I, 2); SM; AR(I, 3); SM; AR(I, 4)
     NEXT I
     PRINT #IV,
     PRINT #IV,
     PRINT #IV, "ALL STATISTICS ARE DETERMINED FOR THE UNIT SIDE"
     CLOSE IV
     END SUB

     SUB FINALE
' A SUBROUTINE TO SOUND THE PROGRAM TERMINATION SIGNAL
'
     NOTE "3C0506"
     NOTE "4D0509"
     NOTE "5E0512"
     NOTE "6F0515"
     END SUB

     SUB MOMENTAL (N, P(), AM, VP, VS, VQ, VR)
' A SUBROUTINE TO COMPUTE THE MOMENTAL DESCRIPTIVE STATISTICS OF
' THE SIMPLE DATA DISTRIBUTION XM().
' THE METHOD IS TAKEN FROM PAGE 30 OF "STATISTICS IN GEOGRAPHY"
' ( SECOND EDITION ) BY DAVID EBDON ( ISBN 0-631-13688-6 ).
```

```
'     ARGUMENTS:
'        N      THE NUMBER OF DATA
'        P()    THE ARRAY OF DATA VALUES
'        AM     THE ARITHMETIC MEAN
'        VP     THE POPULATION STANDARD DEVIATION
'        VS     THE SAMPLE     STANDARD DEVIATION
'        VQ     THE MOMENTAL SKEWNESS
'        VR     THE MOMENTAL KURTOSIS
'
      AM = 0#: FOR I = 1 TO N: AM = AM + P(I): NEXT I: AM = AM / N
      VP = 0#: VS = 0#: VQ = 0#: VR = 0#
      FOR I = 1 TO N
        XD = P(I) - AM
        XX = XD * XD: VP = VP + XX
        XX = XX * XD: VQ = VQ + XX
        XX = XX * XD: VR = VR + XX
      NEXT I
      VS = SQR(VP / (N - 1)): VP = SQR(VP / N)
      VQ = VQ / (N * VP ^ 3): VR = VR / (N * VP ^ 4)
      END SUB

      SUB NOTE (S)
' A SUBROUTINE TO SOUND A NOTE UPON THE COMPUTER SPEAKER
'     ARGUMENT:
'        S     THE NOTE SPECIFIER STRING "IN$NL" e.g. "2B0506"
'              I   THE OCTAVE NUMBER        ( 0-6 )
'              N$  THE NOTE LETTER          ( ABCDEFG )
'              N   THE NOTE NUMBER          ( 0-84 )
'              L   THE LENGTH OF THE NOTE   ( 1-64 )
'
      I = VAL(MID$(S, 1, 1)): N$ = MID$(S, 2, 1)
      N = VAL(MID$(S, 3, 2)): L = VAL(MID$(S, 5, 2))
      PLAY "O" + STR$(I) + "N" + STR$(N) + "L" + STR$(L) + "X" + VARPTR$(N$)
      END SUB

      SUB POLYLOAD (N, NV, JI, JF, JB, JC, X1, Y1, X2, Y2, XB(), YB(), XV(), YV())
' A SUBROUTINE TO DEFINE A PART-POLYGON BY ESTABLISHING ITS
VERTICES
' IN ARRAYS XV(NV) AND YV(NV)
'     ARGUMENTS:
'        N      THE NUMBER OF VERTICES OF THE FULL-POLYGON
'        NV     THE NUMBER OF VERTICES OF THE PART-POLYGON
'        JI     THE INITIALISATION SWITCH
'                0   NO INITIALISATION
'                1      INITIALISATION
'        JF     THE FINALISATION SWITCH
'                0   NO FINALISATION
'                1      FINALISATION
'        JB     THE START  LABEL ON XB,YB
'        JC     THE FINISH LABEL ON XB,YB
'        X1     THE X CO-ORDINATE OF THE INITIAL VERTEX
```

```
'        Y1     THE Y CO-ORDINATE OF THE INITIAL VERTEX
'        X2     THE X CO-ORDINATE OF THE FINAL   VERTEX
'        Y2     THE Y CO-ORDINATE OF THE FINAL   VERTEX
'        XB()   THE ARRAY OF X CO-ORDINATES OF FULL-POLYGON VERTICES
'        YB()   THE ARRAY OF Y CO-ORDINATES OF FULL-POLYGON VERTICES
'        XV()   THE ARRAY OF X CO-ORDINATES OF PART-POLYGON VERTICES
'        YV()   THE ARRAY OF Y CO-ORDINATES OF PART-POLYGON VERTICES
'
      IF JI <> O THEN
        NV = 1: XV(1) = X1: YV(1) = Y1
      END IF
      FOR J = JB TO JC: NV = NV + 1: XV(NV) = XB(J): YV(NV) = YB(J): NEXT J
      IF JF <> 0 THEN
        NV = NV + 1: XV(NV) = X2: YV(NV) = Y2
      END IF
      END SUB

      SUB POLYPOINT (TH, GA, CD, R, A, X, Y)
' A SUBROUTINE TO DETERMINE THE CARTESIAN CO-ORDINATES OF A POINT
' UPON THE BOUNDARY OF A REGULAR POLYGON
'     ARGUMENTS:
'        TH     THE SIDE-SUBTENDED ANGLE
'        GA     THE ANGLE SUBTENDING THE VERTEX RADIUS
'        CD     THE ANGLE PI-GA
'        R      THE VERTEX RADIUS
'        A      THE ANGLE OF THE POINT
'        X      THE POINT X CO-ORDINATE
'        Y      THE POINT Y CO-ORDINATE
'
      M = INT(A / TH): CA = M * TH: BE = A - CA
      RS = R * SIN(GA) / SIN(CD - BE)
      X = RS * COS(A): Y = RS * SIN(A)
      END SUB

      FUNCTION RAN2 (LD) STATIC
' Returns a uniform random deviate between 0.0 and 1.0.
' This subroutine is based upon a single congruential generator
' And a shuffler. It is faster than RAN1 but less various.
' Set LD to any negative value to initialise or reinitialise
' The sequence.
' Adapted from the original QBASIC code of Segment RAN3
' On pp.140-141 of "Numerical Recipes Routines and Examples in BASIC"
' By Julian C Sprott:1991:Cambridge University Press of Cambridge, England
' ( ISBN 0-521-40689-7 )
'
      LM = 714025
      LIA = 1366
      LIC = 150889
      RM = .0000014005112#
      DIM LIR(97)
' As a precaution against misuse, we will always initialise on
```

```
' The first call, even if LD is not set negative.
      IF LD < 0 OR IFF = 0 THEN
        IFF = 1
        LD = (LIC - LD) MOD LM
' Initialise the shuffle table.
        FOR I = 1 TO 97
          LD = (LIA * LD + LIC) MOD LM
          LIR(I) = LD
        NEXT I
        LD = (LIA * LD + LIC) MOD LM
' Utilise LD as an array pointer.
        LIY = LD
      END IF
' Here is where we start except on initialisation.
      J = 1 + INT((97 * LIY) / LM)
      IF J > 97 OR J < 1 THEN PRINT "Abnormal Exit": EXIT FUNCTION
      LIY = LIR(J)
      RAN2 = LIY * RM
      LD = (LIA * LD + LIC) MOD LM
      LIR(J) = LD
      END FUNCTION
```

Appendix Five

Momental Statistics of Polygon Mean Transects
and
Mean Dissected Areas by TRANSECT.BAS
(TRDBCOMP.CAL)

THE MEAN TRANSECT OF A REGULAR POLYGON
ESTIMATED BY COMPUTER AVERAGING
OF INSCRIBED LINES

ALL STATISTICS ARE DETERMINED FOR THE UNIT SIDE

TRDB0064.CAL

THE NUMBER OF TRIALS FOR EACH POLYGON IS 64

NUMBER OF SIDES	MOMENTAL STATISTICS					AREA STATISTICS (MEANS)			
	MEAN	POPULATION SD	SKEWNESS	KURTOSIS	SE	AREA1	AREA2	TOTAL	ANALYTIC
3	0.58406279	0.14997926	-0.21668288	2.58598234	0.01874741	0.17831088	0.25470182	0.43301270	0.43301270
4	0.88703161	0.24586812	-0.92906697	3.57483521	0.03073351	0.38266332	0.61733668	1.00000000	1.00000000
5	1.16791656	0.29098539	-0.81168626	2.50468042	0.03637317	0.62421753	1.09625987	1.72047740	1.72047740
6	1.34595842	0.41612272	-1.00893409	3.32139056	0.05201534	0.89959419	1.69848202	2.59807621	2.59807621
7	1.54043733	0.60542438	-1.02219645	2.85700324	0.07567805	1.21454876	2.41936369	3.63391244	3.63391244
8	1.80942213	0.59074301	-0.91358444	2.71614016	0.07384288	1.58134253	3.24708459	4.82842712	4.82842712
9	2.06034536	0.68494426	-0.94025610	2.81267593	0.08561803	2.00996342	4.17186078	6.18182419	6.18182419
10	2.26114943	0.81111903	-0.99263459	2.95603169	0.10138988	2.46349800	5.23071085	7.69420884	7.69420884
15	3.37198942	1.21058642	-0.81394688	2.38769286	0.15132330	5.47136708	12.17099583	17.64236291	17.64236291
20	4.43679392	1.76590650	-0.90199802	2.58122011	0.22073831	10.28652098	21.28223660	31.56875757	31.56875757
50	10.79535325	4.69792389	-0.90352357	2.62061565	0.58724049	61.54392915	137.13788140	198.68181055	198.68181055
100	21.48892703	9.46865843	-0.89203582	2.59244278	1.18358230	246.68533798	548.82756087	795.51289884	795.51289884

TRDB0256.CAL

THE NUMBER OF TRIALS FOR EACH POLYGON IS 256

NUMBER OF SIDES	MOMENTAL STATISTICS					AREA STATISTICS (MEANS)			
	MEAN	POPULATION SD	SKEWNESS	KURTOSIS	SE	AREA1	AREA2	TOTAL	ANALYTIC
3	0.57919693	0.16417603	-0.06390423	2.63095842	0.01026100	0.17879634	0.25421636	0.43301270	0.43301270
4	0.87300557	0.24186697	-0.75924320	2.81967509	0.01511669	0.37175434	0.62824566	1.00000000	1.00000000
5	1.14287799	0.33998361	-1.03179852	3.32581120	0.02124898	0.61547540	1.10500200	1.72047740	1.72047740
6	1.35684769	0.41803173	-0.85073336	2.79676064	0.02612698	0.90199318	1.69608303	2.59807621	2.59807621
7	1.59473479	0.53847723	-0.94746821	2.89674586	0.03365483	1.25373438	2.38017806	3.63391244	3.63391244
8	1.80660856	0.60121906	-0.77114236	2.44903135	0.03757619	1.53100095	3.29742617	4.82842712	4.82842712
9	2.04192554	0.71620232	-0.81058863	2.53474385	0.04476264	1.96899101	4.21283318	6.18182419	6.18182419
10	2.24187535	0.81506788	-0.81573225	2.52553372	0.05094174	2.52283772	5.17137112	7.69420884	7.69420884
15	3.35935018	1.26206354	-0.77362450	2.35268043	0.07887897	5.62034072	12.02202219	17.64236291	17.64236291
20	4.41538243	1.74877956	-0.74524704	2.33529627	0.10929872	9.91850142	21.65025616	31.56875757	31.56875757
50	10.80126694	4.61725061	-0.72680802	2.26988113	0.28857816	60.64755340	138.03425715	198.68181055	198.68181055
100	21.44880864	9.35442546	-0.71428450	2.24678071	0.58465159	242.01138551	553.50151333	795.51289884	795.51289884

TRDB1024.CAL

THE NUMBER OF TRIALS FOR EACH POLYGON IS 1024

NUMBER OF SIDES	MOMENTAL STATISTICS					AREA STATISTICS (MEANS)			
	MEAN	POPULATION SD	SKEWNESS	KURTOSIS	SE	AREA1	AREA2	TOTAL	ANALYTIC
3	0.57879649	0.17535080	-0.25668703	2.54650137	0.00547971	0.18950899	0.24350371	0.43301270	0.43301270
4	0.86275471	0.25717865	-0.76940752	2.77606727	0.00803683	0.38860850	0.61139150	1.00000000	1.00000000
5	1.10639110	0.35260319	-0.76199532	2.65354996	0.01101885	0.63728631	1.08319109	1.72047740	1.72047740
6	1.33922197	0.44163754	-0.77716905	2.62053891	0.01380117	0.93044177	1.66763444	2.59807621	2.59807621
7	1.57167809	0.54413039	-0.76101355	2.45949211	0.01700407	1.26886974	2.36504271	3.63391244	3.63391244
8	1.77249915	0.64193626	-0.67494365	2.24377967	0.02006051	1.61962006	3.20880706	4.82842712	4.82842712
9	1.99780836	0.74556685	-0.70995268	2.29210266	0.02329896	2.05053248	4.13129171	6.18182419	6.18182419
10	2.20441248	0.83965154	-0.64893017	2.18526590	0.02623911	2.51790815	5.17630069	7.69420884	7.69420884
15	3.25560249	1.34311188	-0.60538659	2.05868305	0.04197225	5.56418118	12.07818174	17.64236291	17.64236291
20	4.29559927	1.83983432	-0.59825377	2.02427565	0.05749482	10.02338512	21.54537246	31.56875757	31.56875757
50	10.49564800	4.83514244	-0.56892180	1.96398605	0.15109820	60.00489980	138.67691075	198.68181055	198.68181055
100	20.80729276	9.83583511	-0.56351654	1.95063487	0.30736985	238.95382950	556.55906935	795.51289884	795.51289884

TRDB4096.CAL

THE NUMBER OF TRIALS FOR EACH POLYGON IS 4096

NUMBER OF SIDES	MOMENTAL STATISTICS MEAN	POPULATION SD	SKEWNESS	KURTOSIS	SE	AREA STATISTICS (MEANS) AREA1	AREA2	TOTAL	ANALYTIC
3	0.57398956	0.17326466	-0.20336369	2.56650013	0.00270726	0.18101355	0.25199915	0.43301270	0.43301270
4	0.86045034	0.25945484	-0.74294728	2.86068776	0.00405398	0.37611577	0.62388423	1.00000000	1.00000000
5	1.10314827	0.35372556	-0.77227062	2.67770904	0.00552696	0.61098508	1.10949233	1.72047740	1.72047740
6	1.33050507	0.44255553	-0.74410676	2.57070239	0.00691493	0.89214569	1.70593052	2.59807621	2.59807621
7	1.54953372	0.53759362	-0.68082518	2.39734840	0.00839990	1.19633520	2.43757724	3.63391244	3.63391244
8	1.76443938	0.63754839	-0.65973722	2.29334217	0.00996169	1.56563614	3.26279098	4.82842712	4.82842712
9	1.97576084	0.73341152	-0.62971583	2.20803164	0.01145955	1.97237470	4.20944949	6.18182419	6.18182419
10	2.18831355	0.82852015	-0.59807554	2.15409138	0.01294563	2.40480278	5.28940606	7.69420884	7.69420884
15	3.22034557	1.32068977	-0.54125317	2.01075938	0.02063578	5.35165086	12.29071205	17.64236291	17.64236291
20	4.25305092	1.80865603	-0.53595426	1.99330489	0.02826025	9.50410772	22.06464985	31.56875757	31.56875757
50	10.37619842	4.75505987	-0.50810200	1.93823916	0.07429781	57.73622144	140.94558911	198.68181055	198.68181055
100	20.56557806	9.66964929	-0.50271933	1.92810819	0.15108827	230.29091205	565.22198680	795.51289884	795.51289884

ALL STATISTICS ARE DETERMINED FOR THE UNIT SIDE

Appendix Six

A Comparison of TRANANAL Numerical and TRANSECT Statistical Solutions For The Mean Transect of Selected Polygons (TRANANCP.CAL)

TRANANCP.TRL

NUMBER OF SIDES	MEAN TRANSECT BY TRANANAL.BAS	MEAN TRANSECT BY TRANSECT.BAS	POLYGON AREA BY TRANANAL.BAS	PERCENTAGE ERROR IN TRANSECTS	PERCENTAGE ABSOLUTE ERROR IN TRANSECTS
3	0.715096397	0.573989562	0.433012702	24.583519235	24.583519235
4	0.869009130	0.860450345	1.000000000	0.994686670	0.994686670
5	1.104867618	1.103148268	1.720477401	0.155858493	0.155858493
6	1.328515670	1.330505074	2.598076211	-0.149522439	0.149522439
7	1.545559577	1.549533718	3.633912444	-0.256473386	0.256473386
8	1.758666113	1.764439385	4.828427125	-0.327201469	0.327201469
9	1.969246197	1.975760844	6.181824194	-0.329728526	0.329728526
10	2.178111850	2.188313553	7.694208843	-0.466190169	0.466190169
15	3.209317720	3.220345571	17.642362911	-0.342443083	0.342443083
20	4.231147617	4.253050917	31.568757573	-0.515002054	0.515002054
50	10.325296619	10.376198416	198.681810548	-0.490563069	0.490563069
100	20.462192772	20.565578064	795.512898844	-0.502710362	0.502710362
				MEAN ERROR (all)	MEAN ERROR (all)
ALL STATISTICS ARE DETERMINED FOR THE UNIT SIDE				1.862852487	2.426158246
				MEAN ERROR (~n=3)	MEAN ERROR (~n=3)
				-0.202662672	0.411852702